Fourth International Conference on Analysis and Applied Mathematics

THE ABSTRACT BOOK

of the conference ICAAM 2018

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Prof. Dr. Charyyar Ashyralyyev

and

Assoc. Prof. Dr. Evren Hincal

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The conference is organized biannually. Previous conferences were held in Gumushane, Turkey in 2012, in Shymkent, Kazakhstan in 2014, and in Almaty, Kazakhstan in 2016. The aim of the International Conference on Analysis and Applied Mathematics (ICAAM) is to bring mathematicians working in the area of analysis and applied mathematics together to share new trends of applications of mathematics. In mathematics, the developments in the field of applied mathematics open new research areas in analysis and vice versa. That is why, we plan to found the conference series to provide a forum for researches and scientists to communicate their recent developments and to present their original results in various fields of analysis and applied mathematics.

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FOREWORD

The Organizing Committee of ICAAM and Near East University, Lefkosa (Nicosia), Mersin 10, Turkey are pleased to invite you to the Fourth International Conference on Analysis and Applied Mathematics, ICAAM 2018. The meeting will be held on September 6-9, 2018 in North Cyprus, Turkey.

The conference is organized biannually. Previous conferences were held in Gumushane, Turkey in 2012, in Shymkent, Kazakhstan in 2014, and in Almaty, Kazakhstan in 2016. The proceedings of ICAAM 2012, ICAAM 2014, and ICAAM 2016 were published in AIP (American Institute of Physics) Conference Proceedings. Near East University is pleased to host the fourth conference which is focused on various topics of analysis and its applications, applied mathematics and modeling.

The conference will consist of plenary lectures, mini symposiums and contributed oral presentations.

The proceedings of ICAAM 2018 were published in AIP Conference Proceedings, too.

Selected full papers of this conference will be published in peer-reviewed journals.

The aim of the International Conference on Analysis and Applied Mathematics (ICAAM) is to bring mathematicians working in the area of analysis and applied mathematics together to share new trends of applications of mathematics. In mathematics, the developments in the field of applied mathematics open new research areas in analysis and vice versa. That is why, we plan to found the conference series to provide a forum for researches and scientists to communicate their recent developments and to present their original results in various fields of analysis and applied mathematics. The Conference Organizing Committee would like to thank our sponsors. The main organizer of the conference is Near East University, Lefkosa (Nicosia), Mersin 10, Turkey. The conference is also supported by Institute of Mathematics and Mathematical Modeling, Almaty, Kazakhstan. We would like to thank Near East University, Turkey and Institute of Mathematics and Mathematical Modeling, Kazakhstan for their support. We also would like to thank to all Invited Speakers, Co-Chairs, Coordinating Committee, International Organizing Committee, International Organizing Committee, and Technical Program Committee Members. With our best wishes and warm regards,

Chairs:

Prof. Allaberen Ashyralyev

Prof. Tynysbek Kalmenov

Contents

ANALYSIS

A note on some fixed point theorems for generalized expansive mappings in cone metric spaces over Banach algebras

Abba Auwalu

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Abstract: In this paper, we prove some fixed point theorems for generalized expansive mappings in non-normal cone metric spaces over Banach algebras. Moreover, we give an example to elucidate our result. Our results are significant extension and generalizations of recent result of B. Jiang et al., (J. Comput. Anal. Appl., 21(6), 1103 - 1114, 2016) and many well-known results in the literature. The following is our main result which generalizes Theorem 3.1 in [1], Theorem 2.2 in [2], Theorem 2.1 in [3], and Theorems 2.3, 2.6 in [4].

Let (Y, ρ) be a complete cone metric space over Banach algebra $\mathcal{A}$, a unit e and $\mathcal{P}$ be a solid cone in $\mathcal{A}$. Let the mapping $\mathfrak{T} : Y \to Y$ be a surjective and satisfies the generalized expansive condition:

$$\rho(\mathfrak{T}y, \mathfrak{T}z) + \vartheta_1[\rho(y, \mathfrak{T}z) + \rho(z, \mathfrak{T}y)] \succcurlyeq \vartheta_2\rho(y, z) + \vartheta_3\rho(y, \mathfrak{T}y) + \vartheta_4\rho(z, \mathfrak{T}z),$$

for all $y, z \in Y$, where $\vartheta_j \in \mathcal{P}$ $(j = 1, 2, 3, 4)$ such that $(\vartheta_1 + \vartheta_2 - \vartheta_4)^{-1} \in \mathcal{P}$, and a spectral radius $\delta[(\vartheta_2 + \vartheta_3 - \vartheta_1)^{-1}(e + \vartheta_1 - \vartheta_4)] < 1$. Then $\mathfrak{T}$ has a fixed point in Y.

Keywords: Cone metric space over Banach algebras, generalized expansive mapping, fixed point, c-sequence

2010 Mathematics Subject Classification: 47H10,54H25

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Cone rectangular metric spaces over Banach algebras and fixed point results of T-contraction mappings

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Abstract: In this paper, we introduce the notion of T-contraction mappings on cone rectangular metric spaces over Banach algebras. Further, we establish the existence and uniqueness of fixed point for such mappings. Our results extend and generalize the Banach contraction principle and Edelstein fixed point theorem given in [1–6], and many recent results in the literature. Moreover, an example to illustrate the main result is also presented.

Keywords: cone rectangular metric spaces, Banach algebras, T-contraction mapping, fixed point.

2010 Mathematics Subject Classification: 47H10, 54H25

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About of the spectrum of regular boundary value problems for one-dimensional differential operators

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Abstract: In this paper, established that every regular non-volterra problem has an infinite spectrum.

On $[0, b]$ we consider the general linear ordinary differential equation

$$L_Q u = u^{(m)}(x) + \sum a_k(x)\, u^{(k)}(x) = f(x) \tag{1}$$

with generalized regular boundary conditions

$$Q_1 u(0) + Q_2 u(b) = 0 \tag{2}$$

where Q_1 and Q_2 are linear operators defined on the traces of the functions $u(0)$ and $u(b)$ and on the traces of its derivatives.

It should be noted that the general form of the regular boundary conditions for equation (1) is given by Kalmenov and Otelbaev (2016).

The question arises. Is there a regular boundary value problem for differential equations that has a finite spectrum?

For the wide class of differential equations Kalmenov and Suragan (2008) established that if a regular boundary value problem has at least one eigenvalue, then the spectrum of this problem is infinite.

In this paper, we assume that $a_k(x) \in C^{\infty}[0, b]$, $\left|\frac{d^m}{dx^m} a_\alpha(x)\right| \le M,\ x \in [0, b], m = 0, 1, 2, ...$ established that every regular non-volterra problem (the spectrum consists of at least one point) has an infinite spectrum.

This research is financially supported by a grant AP05133239 and by the target program BR05236656 from the Science Committee from the Ministry of Science and Education of the Republic of Kazakhstan.

Keywords: One-dimensional differential operators; regular boundary conditions; infinity spectrum; eigenvalue

2010 Mathematics Subject Classification: 34L10, 47E05, 34B09

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An Extended Generalized (p, q)-Extensions for the Apostol type Polynomials

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Abstract: In this report, a new unified generalization for Apostol type polynomials of order α and level m based on (p,q)-numbers is introduced. Then, several new formulas and identities including some algebraic and differential properties are derived. Moreover, multifarious correlations including the well known (p,q)-polynomials such as Apostol type (p,q)-Bernoulli polynomials, Apostol type (p,q)-Euler polynomials, Apostol type (p,q)-Genocchi polynomials (p,q)-Stirling numbers of the second kind, and (p,q)-Bernstein polynomials.

Along this note we mainly use techniques in the paper [1].

Keywords: (p,q)-numbers, Bernoulli polynomials, Euler polynomials, genocchi polynomials, Bernstein polynomials, Stirling numbers of the second kind, Cauchy product

2010 Mathematics Subject Classification: 05A30, 11B68, 11B73.

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On the Existence of Solutions for Fractional Differential Inclusions with Sum and Integral Boundary Conditions

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Abstract: In this paper, by using the method of upper and lower solutions for multifunction, we investigate the existence of solutions for a boundary value problem for fractional differential inclusions with sum and integral boundary conditions.

Keywords: Caputo fractional derivative, Fractional differential inclusion, upper and lower solutions ,Boundary value problem, fixed point, Banach space.

2010 Mathematics Subject Classification: 26A33, 34B15, 34G20

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On the Stablity of the solution of a certain nonlinear elliptic partial differential equations

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Abstract: This report is devoted to the functions which are solution of the nonlinear elliptic partial differential equation $\frac{\overline{f_{\overline{z}}}}{\overline{f}} = a\frac{f_z}{f}$ defined on the unit disc U, a analytic, and $a(U) \subset U$. Such functions are called logharmonic mappings.We study logharmonic mappings which are stable univalent, stable starlike, stable with positive real part , stable close to starlike and stable typically real. We prove that the mappings $f_\lambda = zh(z)\overline{g(z)}^\lambda$ are starlike logharmonic(resp. logharmonic univalent , close to starlike logharmonic, typically real logharmonic) for all $|\lambda| = 1$ if and only if the mappings $\varphi_\lambda = \frac{zh}{g^\lambda}$ are starlike analytic (resp. analytic univalent, close to starlike analytic, typically real analytic) for all $|\lambda| = 1$.

Keywords: Logharmonic mappings, stable, Starlike , Typicallyreal, univalent

2010 MSC: Primary 30C45, 30C55; Secondary 35Q30.

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Characteristic problems for a loaded hyperbolic equation with the wave operator in the principal part

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Abstract: In this paper we consider an equation of the form

$$u_{xx} - u_{yy} + a(x,y)u_x + b(x,y)u_y + c(x,y)u = \lambda \sum_{i=1}^{n} a_i(x,y) D_{0\xi}^{\alpha_i} u\left(\frac{\xi + \alpha(\xi)}{2}, \frac{\alpha(\xi) - \xi}{2}\right), \tag{1}$$

where $\xi = x - y$, λ is the real number, $\alpha_1 < \alpha_2 < ... < \alpha_n = \alpha < 1$, D_{ax}^{l} is the operator of fractional integration as $l < 0$ and fractional differentiation as $l > 0$ of order $|l|$ with the starting point a and the end point x, $\alpha(\xi)$ is a C^2-diffeomorphism, and moreover

$$0 \le \alpha(\xi) \le 1, \xi \in [0,1], \alpha(0) = 0, \tag{2}$$

$\varphi, \psi \in C(\bar{J}), \varphi(0) = \psi(0)$,

$$a, b, c \in C^1(\Omega), a_i \in C(\bar{\Omega}), i = \overline{1,n}, \tag{3}$$

where $\bar{J}$ is the closure of the interval $J = \{(x,y), 0 < x < 1, y = 0\}$.

Assume $\Omega = \{(x,y) : 0 < x + y < 1, 0 < x - y < 1\}$ is the rectangular domain bounded by characteristic equation (1).

Problem G. In the domain Ω find the solution to equation (1) belonging to $C(\bar{\Omega}) \cap C^2(\Omega)$, satisfying the boundary conditions

$$u\left(\frac{x}{2}, \frac{x}{2}\right) = \psi(x), \qquad 0 \le x \le 1, \tag{4}$$

$$u\left(\frac{x}{2}, -\frac{x}{2}\right) = \varphi(x), \qquad 0 \le x \le 1, \tag{5}$$

where $\bar{\Omega}$ is the closure of the domain Ω.

Theorem. *Under the assumptions (2) and (3), the Goursat problem (1), (4) and (5) is always uniquely solvable.*

Keywords: hyperbolic equation, fractional derivative, Goursat problem, wave equation, loaded equation

2010 Mathematics Subject Classification: 35L05

Spectral Properties of one Elliptic Operator in a Punctured Domain

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Abstract: In the work we derive regularized trace formulas which were established in papers of Kanguzhin and Tokmagambetov for the Laplace and m-Laplace operators in a punctured domain with the fixed iterating order $m \in \mathbb{N}$. By using techniques of Sadovnichii and Lyubishkin, the authors in that papers described regularized trace formulae in the spatial dimension $d = 2$. In this note one claims that the formulas are also true for more general operators in the higher spatial dimensions, namely, $2 \leq d \leq 2m$. Also, we give the further discussions on a development of the analysis associated with the operators in punctured domains. This can be done by using so called 'nonharmonic' analysis.

Keywords: Regularized trace formula, m-Laplace operator, punctured domain, non–harmonic analysis

2010 Mathematics Subject Classification: 35J05

Multiperiodic Solutions of the Autonomous Systems with the Operator of Differentiation on the Lyapunov's Vector Field

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Abstract: In this article we consider an autonomous system of a special form with a operator of differentiation whose characteristic system is related with the Lyapunov's systems. The multiperiodicity of zeros of this operator has established. By the methods of the theory of multiperiodic solutions, we prove a theorem on the existence of a unique multiperiodic solution with respect to time variables. The result of the theorem extends to the nonlinear case on the basis of the fixed point method.

The content of the article was the development of a method for investigating the oscillatory solutions of autonomous systems of the form (1). The main essence of the method for investigating the multiperiodic solutions of the system under consideration is a combination of the known methods of [1,2], developed in [3]. In this case, the characteristic system of the operator of differentiation consists of several subsystems of the Lyapunov's class, which generates the multyperiodicity of the solutions of the original system. Note that the developed technique is applicable to a more general system when it contains both time and space variables. The idea of a work is new, which in the simplest case was first tested in [4].

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A generalized Banach contraction principle on cone pentagonal metric spaces over Banach algebras

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Abstract: In this paper, we introduce the concept of cone pentagonal metric spaces over Banach algebras as a generalization of metric space and many of its generalization such as; cone metric space [1,2], cone rectangular metric space [3], and cone pentagonal metric space [4]. Furthermore, we prove a generalized Banach contraction principle in such a space as follows:

Theorem 1.1. *Let $(\mathcal{X}, d)$ be a complete cone pentagonal metric space over Banach algebra $\mathcal{B}$ and S be a non normal solid cone in $\mathcal{B}$. Suppose $T : \mathcal{X} \to \mathcal{X}$ is a mapping that satisfies the following condition:*

$$d(Tx, Ty) \preccurlyeq kd(x, y) \text{ for all } x, y \in \mathcal{X},$$

where $k \in S$ is a generalized Lipschitz constant such that the spectral radius $\delta(k) < 1$. Then T has a unique fixed point x^* in $\mathcal{X}$. Moreover, for any $x \in \mathcal{X}$, the iterative sequence $\{T^i x\}$ $(i \in \mathbb{N})$ converges to x^*.

Keywords: cone pentagonal metric spaces, Banach algebras, c-sequence, contraction mapping principle, fixed point.

2010 Mathematics Subject Classification: 47H10, 54H25

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[1] L.G. Huang, X. Zhang, Cone metric spaces and fixed point theorems of contractive mappings, Journal of Mathematical Analysis and Applications, vol. 332, no. 2, 1468–1476, 2007.
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Monotone operators and the proximal point algorithm in $CAT(0)$ spaces

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Abstract: In this paper, by using products of finitely many resolvents of monotone operators, we propose an iterative algorithm for finding a common zero of a finite family of monotone operators and a common fixed point of a finite family of nonexpansive mappings in $CAT(0)$ spaces. We derive the strong convergence of the proposed algorithm under appropriate conditions.

Keywords: Proximal point algorithm, $CAT(0)$ spaces, monotone operator

2010 Mathematics Subject Classification: 47H09, 47H10

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Generalized weighted composition operators between some spaces of analytic functions

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Abstract: In this work, we give some estimates for the essential norm of generalized weighted composition operator $D^n_{\varphi,u}$ from Bloch type spaces into weighted type spaces. Also we obtain new condition for boundedness and compactness of this operator.

Keywords: generalized weighted composition operators, Bloch type space, essential norm, weighted type spaces.

2010 Mathematics Subject Classification: 30H30, 30H99

On relativistic velocity addition

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Abstract: In Einstein's relativity theory, the sum of the velocities $\mathbf{v}_1 = \langle v_1, 0, 0\rangle$ and $\mathbf{v}_2 = \langle v_2, 0, 0\rangle$ equals to $\mathbf{v} = \langle v, 0, 0\rangle$ with

$$v = \frac{v_1 + v_2}{1 + v_1 v_2/c^2}, \tag{1}$$

where c is the speed of light. This is a special case when the velocities $\mathbf{v}_1$ and $\mathbf{v}_2$ have the same direction. In more general case, when $\mathbf{v}_1$ and $\mathbf{v}_2$ have different directions, the formula is more complicated and does not have ordinary properties such as linearity, commutativity, associativity, etc. In this work we are aimed to develop algebraic operations over the preceding relativistic addition formula which have all ordinary properties.

The method that we will use is based on non-Newtonian calculi initiated in [1]. Previously, non-Newtonian calculus with the exponential reference function, which has the multiplicative and bigeometric modifications, was studied in [2, 3]. In this paper, a suitable non-Newtonian calculus is with the hyperbolic tangent reference function.

We will operate with relative speeds, that is, instead of v we will consider v/c assuming that $|v| < c$. The relative speeds are numbers in $\mathbb{E} = (-1, 1)$. Then for relative speeds v_1 and v_2, Eq. (1) produces the relative speed v by

$$v = v_1 \oplus v_2 = \frac{v_1 + v_2}{1 + v_1 v_2}.$$

This is an addition formula in $\mathbb{E}$.

For other algebraic operations in $\mathbb{E}$, consider hyperbolic tangent function and its inverse defined by

$$\alpha(x) = \tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}},\ x \in \mathbb{R},$$

and

$$\alpha^{-1}(x) = \tanh^{-1} x = \frac{1}{2} \ln \frac{1+x}{1-x},\ x \in \mathbb{E}.$$

It is known that (see, for example, [4])

$$\text{(i)}\quad \mathrm{a} \oplus \mathrm{b} = \alpha(\alpha^{-1}(\mathrm{a}) + \alpha^{-1}(\mathrm{b})) = \frac{\mathrm{a}+\mathrm{b}}{1+\mathrm{ab}}. \tag{2}$$

Based on this, introduce the other algebraic operations on $\mathbb{E}$ by

$$\text{(ii)}\qquad a \ominus b = \alpha(\alpha^{-1}(a) - \alpha^{-1}(b)) = \tfrac{a-b}{1-ab},$$

$$\text{(iii)}\qquad a \otimes b = \alpha(\alpha^{-1}(a) \times \alpha^{-1}(b)) = \frac{\left(\frac{1+a}{1-a}\right)^{\frac{1}{2}\ln\frac{1+b}{1-b}} - 1}{\left(\frac{1+a}{1-a}\right)^{\frac{1}{2}\ln\frac{1+b}{1-b}} + 1},$$

$$\text{(iv)}\qquad a \oslash b = \alpha(\alpha^{-1}(a)/\alpha^{-1}(b)) = \frac{\left(\frac{1+a}{1-a}\right)^{\frac{2}{\ln\frac{1+b}{1-b}}} - 1}{\left(\frac{1+a}{1-a}\right)^{\frac{2}{\ln\frac{1+b}{1-b}}} + 1}.$$

The formulae for $a \otimes b$ and $a \oslash b$ are so that it is difficult to predict them by intuition unless they are not transformed in accordance to hyperbolic tangent reference function.

The set $\mathbb{E}$ with the operations (i)-(iv) transforms all properties of $\mathbb{R}$ to $\mathbb{E}$ making it an ordered field. In particular, the distributivity property can be verified as follows:

$$\begin{aligned}(a \oplus b) \otimes c &= \alpha(\alpha^{-1}(a) + \alpha^{-1}(b)) \otimes c \\ &= \alpha(\alpha^{-1}(\alpha(\alpha^{-1}(a) + \alpha^{-1}(b))) \times \alpha^{-1}(c)) \\ &= \alpha((\alpha^{-1}(a) + \alpha^{-1}(b)) \times \alpha^{-1}(c)) \\ &= \alpha((\alpha^{-1}(a) \times \alpha^{-1}(c) + \alpha^{-1}(b) \times \alpha^{-1}(c)) \\ &= \alpha((\alpha^{-1}(a) \times \alpha^{-1}(c)) \oplus \alpha(\alpha^{-1}(b) \times \alpha^{-1}(c)) \\ &= (a \otimes c) \oplus (b \otimes c).\end{aligned}$$

Moreover, the vector operations (addition, product by scalar, scalar product, vector product, etc.), defined through operations (i)-(iv) of $\mathbb{E}$ have all properties of the same operations defined over $\mathbb{R}$.

Whole ordinary calculus transforms to a new calculus that we can call tanh-calculus, in which the sense of derivative and integral changes. If $f^\star$ denotes the tanh-derivative of f, then

$$f^\star(x) = \alpha\left(\frac{\alpha^{-1}(f(x))'}{\alpha^{-1}(x)'}\right) = \frac{e^{\frac{2f'^2)}{1-f^2(x)}} - 1}{e^{\frac{2f'^2)}{1-f^2(x)}} + 1}. \tag{3}$$

This can be Informally derived as follows:

$$\begin{aligned}((f(y) \ominus f(x)) \oslash (y \ominus x)) &= \alpha\left(\frac{\alpha^{-1}(\alpha(\alpha^{-1}(f(y)) - \alpha^{-1}(f(x))))}{\alpha^{-1}(\alpha(\alpha^{-1}(y) - \alpha^{-1}(x)))}\right) \\ &= \alpha\left(\frac{\alpha^{-1}(f(y)) - \alpha^{-1}(f(x))}{\alpha^{-1}(y) - \alpha^{-1}(x)}\right) \\ &= \alpha\left(\frac{\alpha^{-1}(f(y)) - \alpha^{-1}(f(x))}{y - x} \cdot \frac{y - x}{\alpha^{-1}(y) - \alpha^{-1}(x)}\right) \\ &\to \alpha\left(\frac{\alpha^{-1}(f(x))'}{\alpha^{-1}(x)'}\right).\end{aligned}$$

In a similar way, informally we have

$$\begin{aligned}\bigoplus_{i=1}^{n} f(c_i) \otimes (x_i \ominus x_{i-1}) &= \alpha\left(\sum_{i=1}^{n} \alpha^{-1}(f(c_i))(\alpha^{-1}(x_i) - \alpha^{-1}(x_{i-1}))\right) \\ &= \alpha\left(\sum_{i=1}^{n} \alpha^{-1}(f(c_i))(x_i - x_{i-1})\frac{\alpha^{-1}(x_i) - \alpha^{-1}(x_{i-1})}{x_i - x_{i-1}}\right) \\ &\to \alpha\left(\int_a^b \alpha^{-1}(f(x))\alpha^{-1}(x)'\,dx\right).\end{aligned}$$

Therefore, the tanh-integral is defined by

$$\int_a^b f(x)\,d^\star x = \alpha\left(\int_a^b \alpha^{-1}(f(x))\alpha^{-1}(x)'\,dx\right) = \frac{e^{2\int_a^b \ln\frac{1+f(x)}{(1-f(x))(1-x^2)}dx} - 1}{e^{2\int_a^b \ln\frac{1+f(x)}{(1-f(x))(1-x^2)}dx} + 1}. \tag{4}$$

A removal of $\alpha^{-1}(x)'$ in (3) and (4), produces another simplified modification of tanh-derivative and tanh-integral.

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Keywords: Relativistic velocity, calculus, derivative, integral

2010 Mathematics Subject Classification: 30E20,30E99

On solution uniqueness of the Cauchy problem for a third-order partial differential equation with time-fractional derivative

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Abstract: Consider the equation

$$\left(\frac{\partial^\sigma}{\partial y^\sigma} - \frac{\partial^3}{\partial x^3}\right) u(x,y) = f(x,y), \tag{1}$$

where $\sigma \in (0,1)$, and $\partial^\sigma / \partial y^\sigma$ stands for the fractional derivative of order σ with respect to y. The fractional differentiation is given by the Dzhrbashyan-Nersesyan operator (see [1]) associated with ordered pair $\{\alpha, \beta\}$, i.e.

$$\frac{\partial^\sigma}{\partial y^\sigma} = D_{0y}^{\{\alpha,\beta\}} = D_{0y}^{\beta-1} D_{0y}^{\alpha}, \qquad \alpha, \beta \in (0,1], \quad \sigma = \alpha + \beta - 1, \tag{2}$$

where $D_{0y}^{\beta-1}$ and D_{0y}^{α} are the Riemann-Liouville fractional integral and derivative, respectively (see [2]).

In paper [3], a fundamental solution of equation (1) and a representation for solution of the Cauchy problem

$$\lim_{y\to 0} D_{0y}^{\alpha-1} u(x,y) = \tau(x), \qquad x \in \mathbb{R}, \tag{3}$$

in the domain $\mathbb{R} \times (0,T)$, were constructed.

Here, we prove a uniqueness theorem for the problem (1), (3) in the class of fast-growing functions satisfying an analogue of the Tychonoff condition.

Keywords: fractional derivative, third-order partial differential equation, Tychonoff's condition, Dzhrbashyan-Nersesyan operator, Cauchy problem

2010 Mathematics Subject Classification: 35R11, 35A02

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Existence and uniqueness results for Multiplicative Fractional differential equation with three point integral boundary value problem

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Abstract: Fractional differential equations have been a hot topic of recent years since it found many applications ranging from Biology, Chemistry, Engineering, physics, control theory, Electrical circuits [2]. On the other hand, the concept of multiplicative calculus which was well established in [1] is another active research area since it provides an interesting aspects of the known phenomena of mathematics and provide important achievement in some areas such as Numerical Analysis, Fractional and complex analysis. Furthermore, this topic starts to find many applications in economics , Image processing and functional Analysis. Recently, the multiplicative fractional differentiation and integration have been defining in [3] and in this paper, we focus on the investigation of the existence and uniqueness of multiplicative fractional differential equation with three point boundary value problem below;

$$
\begin{aligned}
{}_c^*D^q x(t) &= f(t, x(t)) \\
x(0) &= 1 \\
x(1) &= \left(\int_0^\eta x(s)^{ds}\right)^\alpha
\end{aligned}
$$

the most important note is that Multiplicative fractional calculus is established for some differential equations with non-linear continuous function. Additionally the multiplicative analogue of the solution of BVP is defined.

Keywords: multiplicative, fractional calculus, three point BVP, Caputo derivative, differential equation.

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Heisenberg's inequality in Morrey spaces

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Abstract: The (classical) Heisenberg inequality is given by

$$\|f\|_{L^2}^2 \lesssim \left(\int_{\mathbb{R}} (x-a)^2 |f(x)|^2 \, dx\right)^{\frac{1}{2}} \left(\int_{\mathbb{R}} (\xi-\alpha)^2 |\hat{f}(\xi)|^2 \, d\xi\right)^{\frac{1}{2}}, \tag{1}$$

where $f \in L^2(\mathbb{R})$, $a, \alpha \in \mathbb{R}$, and $\hat{f}$ denotes the Fourier transform of f. There are several generalizations of (1) in Lebesgue spaces over $\mathbb{R}^n$ (see [1–4]). In this talk we discuss a generalization of the Heisenberg inequality in Morrey spaces. We combine some Hardy type inequality and an interpolation inequality for the fractional power of the Laplacian to prove our result and to remove some restrictions in [2]. Our proof follows the idea in [3]. This is a joint work with Hendra Gunawan (Bandung Institute of Technology), Eiichi Nakai (Ibaraki University), and Yoshihiro Sawano (Tokyo Metropolitan University) (see [5]).

Keywords: Imaginary power of Laplace operators, fractional power of Laplace operators, interpolation inequality, Hardy's inequality, Heisenberg's inequality, Morrey spaces.

2010 Mathematics Subject Classification: 42B20,42B35

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Applied Mathematics

About convergence of difference approximations for optimization problems described by elliptic equations with mixed derivatives and unbounded non-linearity

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Abstract: This report is devoted to an optimal control problem for nonlinear elliptic equation with mixed derivatives and unbounded non-linearity, namely, minimizing of a functional $J(g) = \int\limits_{\Omega} |u(x;g) - u_0(x)|^2 d\Omega$, on solutions $u(g)$ to problem

$$-\sum_{\alpha=1}^{2} k_{\alpha\alpha}(x)\frac{\partial^2 u}{\partial x_\alpha^2} - 2k_{12}(x)\frac{\partial^2 u}{\partial x_1 \partial x_2} + q(u)u = f(u),\ x \in \Omega;$$
$$u(x) = 0, \qquad x \in \partial\Omega = \Gamma,$$

obeying all admissible controls

$$g = \big(k_{11}, k_{22}, k_{12}\big) \in U = \Big\{k_{\alpha\beta} \equiv g_{\alpha\beta} \in W_\infty^1(\Omega), \alpha, \beta = 1, 2 :$$
$$\nu \sum_{\alpha=1}^{2} \xi_\alpha^2 \le \sum_{\alpha,\beta=1}^{2} k_{\alpha\beta}(x)\xi_\alpha\xi_\beta \le \mu \sum_{\alpha=1}^{2} \xi_\alpha^2,\ k_{\alpha\beta}(x) = k_{\beta\alpha}(x),$$
$$\forall x \in \Omega, \xi \neq 0, \xi \in \mathbb{R}^2, \left|\frac{\partial k_{\alpha\beta}}{\partial x_1}\right| \le R_1, \left|\frac{\partial k_{\alpha\beta}}{\partial x_2}\right| \le R_2, \alpha, \beta = 1, 2\Big\},$$

where $u_0^{(1)} \in W_2^1(\Omega_1)$ is a given function, $q(\eta)$, $f(\eta)$ are given functions of η. Suppose that the conditions imposed on the coefficients of the state equation are satisfied only in the vicinity of the exact solution values that indicates the presence of non-linearities of the unbounded growth.

Present work is dedicated to the development and study of difference approximations for the given class of optimization problems, namely, study of issues, such as the convergence of approximations with respect to the state, functional, and control (see, e.g., [1], [2] and the references therein).

Keywords: Elliptic equations with mixed derivatives, optimal control, objective functional, unbounded non-linearity, difference method

2010 Mathematics Subject Classification: 49J20, 35J61, 65N06

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Well-posedness and energy decay of solutions to a Lamé system under boundary fractional derivative controls

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Abstract: In this paper, we investigate the existence and decay properties of solutions for the initial boundary value problem of the Lamé system with boundary control conditions of fractional derivative type:

$$(P) \qquad \begin{cases} u_{tt} - \mu\Delta u - (\mu+\lambda)\nabla(div\ u) = 0 \quad \text{in } \Omega\times(0,+\infty) \\ u = 0 \quad \text{in } \Gamma_0\times(0,+\infty) \\ \mu\frac{\partial u}{\partial \nu} + (\mu+\lambda)(div\ u)\nu = -\gamma\partial_t^{\alpha,\eta}u \quad \text{in } \Gamma_1\times(0,+\infty) \end{cases}$$

where $u = (u_1, u_2, ..., u_n)^T$, ν stands for the unit normal vector of Ω pointing towards the exterior and μ, λ, γ are positive constants. The notation $\partial_t^{\alpha,\eta}$ stands for the generalized Caputo's fractional derivative of order α with respect to the time variable. It is defined as follows

$$\partial_t^{\alpha,\eta}w(t) = \frac{1}{\Gamma(1-\alpha)}\int_0^t (t-s)^{-\alpha}e^{-\eta(t-s)}\frac{dw}{ds}(s)\,ds \qquad \eta \geq 0.$$

The system is finally completed with initial conditions

$$u(x,0) = u_0(x), \quad u_t(x,0) = u_1(x).$$

Our purpose in this paper is to give a global solvability in Sobolev spaces and energy decay estimates of the solutions to the problem (P).

Keywords: Lamé system, Fractional feedback, Polynomial stability, Semigroup theory.

2010 Mathematics Subject Classification: 35B40, 74D05, 47D03

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On approximation of first order derivatives of complex-valued functions by finite differences

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Abstract: Boundary value problems for partial differential equations involving complex valued functions have important applications in a broad sense ([1–5]). The theory of finite difference method in case of real valued function and its applications to solve boundary value problems for partial differential equations is described in [6]. Complex step method for computing derivatives of real valued functions by introducing a complex step in a strict sense is considered in [7, 8] (see also references therein).

In this presentation, we generalize the well known finite difference method to compute derivatives of real valued function to approximate of complex derivatives w_z and $w_{\overline{z}}$ for complex valued function w. Exploring different combinations of terms, we derive several approximations to compute the first order derivatives of complex valued function w. The first, second, third and fourth order of accuracy finite differences to calculate derivatives are studied. Error analyses in test examples are carried out by using Matlab program.

Keywords: finite difference, complex-valued function, approximation, error.

2010 Mathematics Subject Classification: 30E05, 30E10

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Some Parseval-Goldstein type identities for the generalized Macdonald and Hankel transforms

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Abstract

In the present paper the authors show that iterations of the generalized Hankel transform with the generalized Macdonald transform is a constant multiple of the generalized Widder transform. Using these iteration identities, several Parseval-Goldstein type theorems of these transforms are given. By making use of these results a number of new Goldstein type exchange identities are obtained. The identities proven in this paper are shown to give rise to useful corollaries for evaluating infinite integrals of special functions. Some examples are also given as illustration of the results presented here.

Keywords: Hankel transforms, Macdonald transforms, Widder transforms, Laplace transforms, Parseval-Goldstein type identities, Goldstein type exchange identities.

2010 AMS Mathematics Subject Classification:

Primary 44A10, 44A15, secondary 33C10, 44A35

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A study on Heron triangles and difference equations

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Abstract: In this paper, we investigate the relationship between Heron triangles formed by consecutive ordered triple $(un-v, un, un+v) \in \mathbb{Q}^3$ and Difference Equation where u, n, v are rational numbers.

Keywords: Heron Triangle, Difference Equation, Consecutive Trios.

Linear equations with two variables root of real numbers indicates a line. However find the integer roots of same equation requires a completely different approach. Similar to obtain the length of the side and the area which are triangles with rational of the family of triangles requires a different method. This paper produces an infinite family of triangles with rational sides and rational areas and, in fact, an infinite family of triangles with integer sides and integer areas. The results are quite attractive and open a door to a further study of these triangles, both geometrically and number-theoretically.

In [1], Buchholz and Rathbun (1997) studied an infinite set of Heron triangles with two rational medians. In [2], Buchholz and Rathbun (1998) presented a proof that there exist infinitely many rational sided triangles with two rational medians and rational area. These triangles correspond to rational points on an elliptic curve of rank one. They also displayed three triangles (one previously unpublished), which do not belong to any of the known infinite families. In [3], Sastry applied a technique in his study that uses a triple of integers that can represent the lengths of the sides of a triangle to generate a family of Heron triangles. In [7], Chisholm and MacDougall examined the remaining six configurations, which were left unsolved by Buchholz [R.H. Buchholz, Perfect pyramids, Bull. Austral. Math. Soc. 45 (1991) 353–368] from the fifteen configurations for tetrahedra having integer edges and volume, by restricting attention to those with two or three different edge lengths, for integer volume, completely solving all but one of them. In [9], Ionascu et al. studied the function $H(a,b)$, which associates to every pair of positive integers a and b the number of positive integers c such that the triangle of sides a, b and c is Heron, i.e., it has integral area. In particular, they proved that $H(p,q) \leq 5$ if p and q are primes, and that $H(a,b) = 0$ for a random choice of positive integers a and b.

In this study, it is investigated the n which is rational numbers, so that obtained consecutive triples that makes a triples of Heron against each value given to v in $(un(k)-v, un(k), un(k)+v) \in \mathbb{Q}^3$ which and u, v are rational numbers, can be obtained from the equation $n(k) = \frac{1}{u}\sqrt{3m^2(k)+4^2}$, where the where $m(k)$ is derived from $m(k) = \frac{1}{\sqrt{3}}v\left\{\left(2+\sqrt{3}\right)^k - \left(2-\sqrt{3}\right)^k\right\}$, $k = 0, 1, 2, \ldots$, which are the solutions of the linear difference equation $m(k+2) - 4m(k+1) + m(k) = 0$ with the initial conditions $m(0) = 0$ and $m(1) = 2v$.

Theorem 2.1. *The Heronian triangles in the form* $(un-1, un, un+1)$ *are obtained from difference equation* $m(k+2) - 4m(k+1) + m(k) = 0$ *, initial conditions of which are* $m(0) = \frac{2}{u}$ *and* $m(1) = \frac{4}{u}$ *. The area of such triangles are obtained from the formula* $\triangle_k = \frac{\left\{(2+\sqrt{3})^{2k} - (2-\sqrt{3})^{2k}\right\}}{4}$ *where* $k = 0, 1, 2, \ldots$.

Theorem 2.2. *The Heronian triangles in the form* $(un-2, un, un+2)$ *are obtained from difference equation* $m(k+2) - 4m(k+1) + m(k) = 0$ *, initial conditions of which are* $m(0) = \frac{4}{u}$ *and* $m(1) = \frac{8}{u}$ *. The area of such triangles are obtained from the formula* $\triangle_k = \sqrt{3}\left\{\left(2+\sqrt{3}\right)^{2k} - \left(2-\sqrt{3}\right)^{2k}\right\}$ *where* $k = 0, 1, 2, \ldots$.

Theorem 2.3. *The Heronian triangles in the form* $(un - v, un, un + v)$ *are obtained from difference equation* $m(k+2) - 4m(k+1) + m(k) = 0$ *, initial conditions of which are* $m(0) = 0$ *and* $m(1) = 2v$ *. The area of such triangles are obtained from the formula*
$\triangle_k = \sqrt{3}\left\{\left(2+\sqrt{3}\right)^{2k} - \left(2-\sqrt{3}\right)^{2k}\right\}$ *where* $k = 0, 1, 2, \ldots$

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An inverse problem of heat conduction in a degenerating domain

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Abstract: This report is devoted to an inverse problem of finding a source parameter $\lambda(t)$ and $u(x,t)$ in following heat equation:

$$u_t(x,t) = u_{xx}(x,t) - \lambda(t)u(x,t), \quad 0 < x < t, t > 0, \tag{1}$$

with homogeneous initial condition

$$u(x,0) = 0, \tag{2}$$

and the boundary condition

$$u(x,t)|_{x=0} = 0, \quad u(x,t)|_{x=t} = 0, \tag{3}$$

subject to the overspecification

$$\int_0^t u(x,t)dx = E(t), \quad E(0) = 0, \tag{4}$$

where the function $E(t)$ is given.

In this paper we found a nontrivial solution of the inverse problem for the heat equation in a degenerate domain that satisfies the integral condition (4) and found the lambda parameter. It was also shown that the found nontrivial solution is a bounded function for $\forall t > 0$.

Throughout this note we mainly use techniques from our works [1], [2].

Keywords: An inverse problem, heat equation, degenerating domain, integral condition.

2010 Mathematics Subject Classification: 35J05, 35J08, 35J25

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The Solution of One Dimensional Fractional Hyperbolic Partial Differential Equations Using Sumudu Transform

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Abstract: In this study, we propose a new algorithm to find exact solution of one dimensional hyperbolic partial diferential equations.The Sumudu transform method can be used to get exact solutions of one dimensional fractional hyperbolic partial differential equations.The comparison of variational iteration method and these method is presented.

Key Words: Fractional hyperbolic equation; initial boundary value problems; sumudu transform method, iteration method

The Gauss Decomposition for the Group $SL(2,\mathbb{R})$ and Casimir Operator

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Abstract: We considered the Gauss decomposition for the Lie group $SL(2,\mathbb{R})$. The Casimir operator of the group and continuous state solution of the system were obtained. The Laplace-Beltrami operator on the surface of the manifold was given and the relation between the Casimir operator and the Laplace-Beltrami operator was observed.

Keywords: Lie group, Gauss decomposition, Casimir operator

2010 Mathematics Subject Classification: 22Exx, 57S25, 22E70

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Investigation of the utility function in the problem of consumer choice

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Abstract: Interesting and useful for studying is the general model of consumer choice with an arbitrary number of products and a general objective objective function.

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Travelling wave solutions for the two dimensional Hirota system of equations

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Abstract: In this talk, we investigate the two dimensional Hirota system of equations by next form

$$
\begin{aligned}
&(1) \qquad & iq_t + \alpha q_{xy} + i\beta q_{xxy} - vq + i(wq)_x &= 0, \\
&(2) \qquad & v_x + 2\alpha\delta(|q|^2)_y - 2i\beta\delta(q^*_{xy}q - q^*q_{xy}) &= 0, \\
&(3) \qquad & w_x - 2\beta\delta(|q|^2)_y &= 0,
\end{aligned}
$$

where $q(x, y, t)$ is complex function, $v(x, y, t), w(x, y, t)$ are real functions, α, β, δ are constants.

The equations (1)-(3) were presented in work [1], [2]. We apply the extended tanh method for the two dimensional Hirota system of equations in order to obtain new exact solutions. The extended tanh method constructs traveling wave solutions in terms of a hyperbolic tangent functions [3]. New families of solitary wave solutions and periodic solutions are obtained for the two dimensional Hirota system of equations.

Keywords: Hirota system, extended tanh method, traveling wave solution

2010 Mathematics Subject Classification: 35C07, 35Q51

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Asymptotic expansion of the solution of the nonlocal boundary value problem with initial jumps for singularly perturbed integro-differential equation

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Abstract: This report is devoted to the nonlocal boundary value problem with initial jumps for the integro-differential equation with a small parameters in the highest derivatives [1]:

$$L_\varepsilon y \equiv \varepsilon^2 y''' + \varepsilon A_0(t)y'' + A_1(t)y' + A_2(t)y = F(t) + \int_0^1 \sum_{i=0}^{1} H_i(t,x)y^{(i)}(x,\varepsilon)dx, \tag{1}$$

with the boundary conditions

$$y(0,\varepsilon) = \alpha, \quad y'(0,\varepsilon) = \beta, \quad y(1,\varepsilon) = \gamma + \int_0^1 \sum_{i=0}^{1} a_i(x)y^{(i)}(x,\varepsilon)dx, \tag{2}$$

Here $\varepsilon > 0$ is a small parameter, α, β, γ are known constants.

In this paper we consider the case when the roots $\mu_i(t), \;\; i = 1,2$ of the additional characteristic equation $\mu^2(t) + A_0(t)\mu(t) + A_1(t) = 0$ have opposite signs. Asymptotic expansion of the solution of the nonlocal boundary value problem with initial jumps for linear singularly perturbed third-order integro-differential equations is constructed with any degree of accuracy with respect to a small parameter.

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Keywords: singular perturbations, small parameter, asymptotic expansion

2010 Mathematics Subject Classification: 34B05, 34B10, 34E05, 34E10

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Vertex magic total labeling of selected trees

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Abstract: A VMT labeling of a graph with v vertices and e edges is defined as a one-to-one map taking the vertices and edges onto the integers 1,2,...,v+e with the property that the sum of the label on a vertex and the labels on its incident edges is a constant independent of the choice of vertex. In MacDougall et. al. [1] along with many interesting results on VMT graphs, the authors observed that, for the given graph a VMT labeling exists if there is not much variation among the degrees of the vertices. In this study further VMT labelings of a class of simple trees with vertex degrees less than four will be given.

Keywords: Vertex magic, total labeling, trees, graphs

2010 Mathematics Subject Classification: 05C78, 05C07

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Stability of basis property of a type of problems with nonlocal perturbation of boundary conditions

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Abstract:

We consider the problem of determining the position of the conjugate points of a combined trajectory, ensuring the conditions of tangency, continuity, and equality of the radius of curvature. When the arcs of a circular and elliptical trajectory are conjugated at the junction where there is a common tangent, a jump in the centrifugal force occurs due to the inequality of the radii of curvature, which will lead to a shock / 1-2 /.

In order to avoid a jump in curvature, the method of inserting a second-order smoothing curve (the transition cone) is applied. There are known methods of continuous, smooth connection of trajectories, which include contours, splines. With such conjugations, the conditions of tangency and continuity of two arcs of curves are satisfied, but the equality condition for the radii of curvature of these curves at the junction point is not satisfied. A mathematical condition for unstressed conjugation of trajectories was defined. Setting the starting point of the connection and using the condition of smoothness (second-order smoothness), one can determine the end point of the transition curve. The process is proposed to be modeled by a rocking mechanism, which allows to visually, quickly and effectively determine the position of the finish point on the circle. Using the method of determining the position of the conjugate points based on the kinematics of the rocking mechanism, it is possible to smoothly join conical arcs satisfying the conditions of continuity, tangency and equality of curvature and to create on their basis new models of treadmills (trajectories) from conical arcs that allow eliminating unwanted impact effects. **2010 Mathematics Subject Classification:** 35J05, 35J08, 35J25

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On the stability estimation of finite-difference analogue of the integral geometry problem with a weight function

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Abstract: The problems of integral geometry are to find the functions, which are determined on certain variety, through its integrals on certain set of subvarities with lower dimension.

Additionally, the problems of integral geometry are correlated with various solutions (data interpretation objectives of exploration seismology, electro-exploration, acoustics, and inverse problems of kinetic equations, widely used in plasma physics and astrophysics). In recent years, the studies on problems of integral geometry have critical significance for tomography, which is intensively developing scientific - technic pillar that has several applications in medicine and industry. Therefore, development of various solution methods for the integral geometry problems is actual issue.

One of the stimuli for studying such problems is their connection with multidimensional inverse problems for differential equations [1]. In some inverse problems for hyperbolic equations were shown to reduce to integral geometry problems and, in particular, a problem of integral geometry was considered in the case of shift-invariant curves. Mukhometov [2] showed the uniqueness and estimated the stability of the solution of a two-dimensional integral geometry on the whole. His results were mainly based on the reduction of the two-dimensional integral geometry problem.

In this work finite - difference analogue of the two-dimensional problem of integral geometry with a weight function are studied. The stability estimate for the considered problem are obtained.

Keywords: integral geometry problem, finite - difference problem, solution, stability.

2010 Mathematics Subject Classification: 65N06, 65N20, 65N21

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Six Point Implicit Methods for the Pure Second Derivatives of the Solution of First Type Boundary Value Problem for One Dimensional Heat Equation

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Abstract: We construct six point implicit difference boundary value problem for the first derivative of the solution $u(x,t)$ of the first type boundary value problem for one dimensional heat equation with respect to the time variable t. Furthermore, for the second order pure derivatives of $u(x,t)$ special six point implicit difference boundary value problems are proposed. A uniform approximation of the order $O\left(h^2+\tau^2\right)$(second order accurate in the spatial varıable x and second order accurate in time t) where h is the step size in spatial variable x and τ is the step size in time is achieved. It is assumed that the initial function belongs to the Hölder space $C^{10+\alpha}$, $0<\alpha<1$, the heat source function is from the Hölder space $C_{x,t}^{8+\alpha,4+\frac{\alpha}{2}}$, the boundary functions are from $C^{5+\frac{\alpha}{2}}$, and between the initial and the boundary functions the conjugation conditions of orders $q=0,1,2,3,4,5$ are satisfied. Theoretical results are justified by numerical examples.

Peculiarities of Constructing an Optimal Vector Distributed Control with Nonlinear Optimization of OscillationProcesses

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Abstract: Nonlinear optimization problem is investigated for oscillation processes described by Fredholm integro-diïfferential equations in partial derivatives when the function of the external source nonlinearly depends on vector distributed control. It is established that, the optimal control procedure is greatly simplified with vector control. Algorithm is developed for constructing a complete solution of the nonlinear optimization problem.

Keywords: Boundary value problem, generalized solution, vector distributed control, functional, property of equal relations, system of nonlinear integral equations

2010 Mathematics Subject Classification: 49L20, 35K20

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Processing of big data in the detection of geochemical anomalies of rare-earth metal deposits

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Abstract: The paper presents a numerical method for processing a large amount of data in detection of anomalies of the chemical elements distribution in polymetallic deposits. Mathematically, the problem reduces to solving the Fredholm integral equation of the first kind [1-3] for large number of different right-hand parts, while the kernel remains unchanged.

$$\int_a^b K(x,s)u(s)ds = f(x), \qquad x \in [a,b]. \tag{1}$$

The algorithm consists of two stages: at the first stage a number of problems are solved, for which knowing the right part is not required, in the second stage, the previously obtained data are used to find solutions of the integral equation for different right-hand sides. This approach allows the data processing "in-situ" and to define prospects of the established abnormal areas, allocate the overriding areas for geological exploration. The algorithm is tested on model and real data.

Keywords: ill-posed problem, inverse problem, integral Fredholm equation of the first kind, kernel, conjugate equation, Fourier series, Fourier coefficients, regularization Lavrentieva, geochemical parameters, chemical elements

2010 Mathematics Subject Classification: 45B05, 86A22, 49N45, 65Q05

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Iterative Method for Solving Non-Linear Navier-Stokes Equations in Complex Domains Taking Into Account Boundary Conditions with Uniform Accuracy

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Abstract: In this report the Navier-Stokes equations describing the motion of viscous incompressible fluid in a bounded domain is considered. Method of fictitious domains is applied for approximate solution of the problem taking into account boundary conditions with uniform accuracy. The original problem is reduced to solving an auxiliary problem corresponding to the method of fictitious domains.

A theorem on the existence of the generalized solution to the auxiliary problem is proved. In addition, the convergence of the solution of the auxiliary problem to the solution of the original problem is proved.

Next, a difference scheme of the second order of approximation is constructed for the auxiliary problem. For a numerical solution of this difference problem, a special iterative method is constructed that determines approximate solutions on the boundary with uniform accuracy for a limited number of arithmetic operations.

To develop a new numerical implementation algorithm, the idea of the fictitious unknowns method with a two-step iterative process and a method for solving the Poisson difference equation in a square with the right-hand side different from zero only at nodes that are a distance of the order of the grid distance from a given piecewise smooth curve are used.

Keywords: method of fictitious domains, Navier-Stokes equations, numerical solution, auxiliary problem, convergence.

2010 Mathematics Subject Classification: 65M85, 65M06, 65N22

A third order of accuracy difference scheme for Bitsadze-Samarskii type multi-point overdetermined elliptic problem

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Abstract: Let us $k_i, \lambda_i, 1 \leq i \leq q; \lambda_0, T, \alpha$ are known nonnegative real numbers such that

$$\begin{aligned} &\sum_{i=1}^{q} k_i = 1, k_i \geq 0, i = 1, \ldots, q, \\ &0 < \lambda_1 < \lambda_2 < \ldots < \lambda_q < T, \lambda_0 \in (0,T), 0 < \alpha < 1. \end{aligned} \tag{1}$$

Consider the following multipoint multidimensional elliptic problem with overdetermination

$$\begin{cases} -v_{tt}(t,x) - \sum\limits_{r=1}^{n} (a_r(x) v_{x_r}(t,x))_{x_r} + \sigma v(x) = g(t,x) + p(x), \\ x = (x_1, \ldots x_n) \in \Omega, 0 < t < T, \\ v(0,x) = \phi(x), v(T,x) - \sum\limits_{i=1}^{q} k_i v(\lambda_i, x) = \eta(x), \\ v(\lambda_0, x) = \zeta(x), x \in \overline{\Omega},\ v(t,x) = 0, x \in S, \end{cases} \tag{2}$$

where $\Omega = (0,\ell)^n$ is the open cube in R_n with boundary S, $\overline{\Omega} = \Omega \cup S$ and nonnegative real numbers σ, λ_0, λ_i, k_i, $1 \leq i \leq q$ are known, smooth functions a_r, ϕ, η, ζ, and f are given on Ω, $a_r(x) > 0$, $\forall x \in \Omega$. In [1], well posedness of problem 2 was established. A first and a second order difference schemes for its approximately solution were constructed. In [2], overdetermined problem for the multi-dimensional elliptic equation with Neumann boundary condition was investigated. The papers [3–5] are devoted to high order approximations of overdetermined elliptic problems without nonlocal boundary conditions.

In the present work, we construct a third order of accuracy difference scheme for problem 2 and establish stability, almost coercive stability and coercive stability estimates for its solution.

Keywords: nonlocal boundary condition,inverse elliptic problem, well-posedness, stability, coercive stability, overdetermination.

2010 Mathematics Subject Classification: 35N25, 35J67

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Multi-Objective mean-CVaR model under VG process on evaluating Portfolio efficiency

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Abstract: In this study, we evaluate the portfolio performance by Data Envelopment Analysis (DEA) as a nonparametric efficiency analysis tool. Our model describes the dynamics of assetsâ2122 log prices by a stochastic process which is named Variance-Gamma (VG) process [1]. Risk measure of our model is Conditional Value at Risk (CVaR)[2]. Therefore, the model is in mean-CVaR framework under VG process. Conventional DEA models consider mainly nonnegative data. However, in real data world, inputs and outputs may as well take negative values. So, our proposed model is the Range Directional Measure-like model that can take positive and negative values[3]. Finally, we present a case study of the stock market to demonstrate the applicability of the proposal model. For estimating the parameters in the model we use Monte Carlo approach and a nonlinear programming technique.

2010 Mathematics Subject Classification: 91G10,91G60

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Stationary boundary-value problems of the anisotropic elastic medium dynamics

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Abstract: The solution of boundary value problems of dynamics of elastic media is one of the topical problems of mathematical physics, because by them is described a wide class of various geophysical processes, dynamic processes in materials and constructions that are widely used in bilding and engineering. The most are developed the mathematical methods for investigation of boundary value problems of the dynamics of isotropic elastic media. Anisotropic media have been little studied, which is connected with the complexity of constructing solutions of equations that are usually solved on the basis of numerical difference or finite element methods.

Here we consider the solution of problems of stationary oscillations with a frequency ω for anisotropic elastic media, which for complex amplitudes is described by a system of equations of the form

$$L_{ij}\left(\partial_x, -i\omega\right) u_j(x) + G_i(x) = 0, \tag{1}$$

$$L_{ij}\left(\partial_x, -i\omega\right) = C_{ij}^{ml}\partial_m\partial_l + \delta_{ij}\rho\omega^2, i, j, m, l = \overline{1, N} \tag{2}$$

Here $u_{i,j} = \partial u_i/\partial x_j$, δ_{ij} is Kronecker symbol, ρ is the density of the medium, G_i are the components of the mass force, C_{ij}^{ml} is the matrix of elastic constants, which has symmetry properties with respect to permutation of the indexes $C_{ij}^{ml} = C_{ij}^{lm} = C_{ji}^{ml} = C_{ml}^{ij}$ (everywhere the summation over repeated indices in the indicated range is assumed).

Two boundary value problems for complex amplitudes of displacements of the medium, which belong to the class of boundary value problems for elliptic systems of equations, are given. To solve them we use the method of generalized functions that allows the initial boundary value problem to lead to the solution of a system of differential equations in the space of generalized functions with a certain right-hand side from the class of singular generalized functions of the type of simple and double layers and to use the properties of fundamental solutions of such systems for constructing its generalized solutions.Considered the fundamental solutions, their symmetry properties and asymptotics. A generalization of the Somilyana formula for displacements is obtained which allows by known displacements and stresses on the boundary of body one to find the displacement amplitudes inside it. The analogue of Gauss's formula is presented, which determines the characteristic function of the domain of definition through surface integrals of the stress tensor. On its basis singular boundary integral equations for solving the boundary value problems are constructed.

This work was supported by the Ministry of education and science of Republic of Kazakhstan by Grant No.AP05135494.

Keywords: anisotropic medium, elasticity, fundamental solutions, boundary value problems, stationary oscillations, boundary integral equations

2010 Mathematics Subject Classification: 74B05, 74E10

On the system of rational difference equations

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Abstract: In this paper, we study the following system of difference equations for $n \in \mathbb{N}_0$

$$\left\{\begin{array}{l} x_{n+1} = \frac{x_{n-1}}{y_n x_{n-1} - 1}, \\ y_{n+1} = \frac{y_{n-1}}{x_n y_{n-1} - 1}, \\ z_{n+1} = x_n y_n z_{n-1}, \end{array}\right. \tag{1}$$

where x_0, x_{-1}, y_0, y_{-1}, z_0, z_{-1} real numbers such that $y_0 x_{-1} \neq 1$ and $x_0 y_{-1} \neq 1$.

Theorem 2.4. *Let $y_0 = a$, $y_{-1} = b$, $x_0 = c$, $x_{-1} = d$, $z_0 = e$, $z_{-1} = f$ be real numbers such that $ad \neq 1$ and $cb \neq 1$. Let $\{x_n, y_n, z_n\}$ be a solution of the system* (1). *Then all solutions of* (1) *are*

$$x_n = \left\{\begin{array}{ll} \frac{d}{(ad-1)^n}, & \text{n} - \text{odd}, \\ c(cb-1)^n, & \text{n} - \text{even} \end{array}\right. \tag{2}$$

$$y_n = \left\{\begin{array}{ll} \frac{b}{(cb-1)^n}, & \text{n} - \text{odd}, \\ a(ad-1)^n, & \text{n} - \text{even} \end{array}\right. \tag{3}$$

$$z_n = \left\{\begin{array}{ll} (ca)^n f[(cb-1)(ad-1)]^{\sum\limits_{i=0}^{n-1} i}, & \text{n} - \text{odd}, \\ \frac{d^n b^n e}{[(ad-1)(cb-1)]^{\sum\limits_{i=1}^{n} i}}, & \text{n} - \text{even} \end{array}\right. \tag{4}$$

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Special Functions and Heat Polynomials for the Solution of Free Boundary Problems

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Abstract: The method of special functions and heat polynomials for the solution of problems for the heat equation in domains with moving boundaries is developed for the generalized and axisymmetric heat equations. The corresponding generating and associated functions are introduced. Examples of application of this method for the solution of some problems are presented.

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Numerical solution of stochastic mixed Volterra-Fredholm integral equations driven by space-time Brownian motion via two-dimensional block pulse functions

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Abstract: The goal of this presentation is to give useful method for solving problems formulated by two dimensional stochastic integral equations driven by space-time white noise. Typically, some stochastic problems require numerical methods to obtain a solution and therefore is focused on stochastic integral and partial differential equations to construct a reliable and efficient computational methods.

Since solution of parabolic partial differential equations driven by space-time white noise leads to a stochastic integral equations, we introduce a new method by two dimensional block pulse functions and their operational integration matrix to transform a stochastic mixed Volterra-Fredholm integral equation to a system of algebraic equations. The benefit of this method is lower cost of setting up the system of equations without any integration. So, the computational cost of operations is low. The method is applied to test examples to illustrate the accuracy and implementation of the method.

Keywords: Two-dimensional block pulse functions; Operational matrix; Stochastic operational matrix; Stochastic mixed Volterra-Fredholm integral equations; Space-time white noise; Brownian sheet.

2010 Mathematics Subject Classification: Primary: 65C30, 60H35, 65C20; Secondary: 60H20, 68U20, 60H15.

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The construction of a solution of a related system of the Laguerre type

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Abstract: The systems of the Laguerre type, which were obtained from the system of Horn by direct selection of parameters as well as by exponential transformation, are considered. For the construction of their normal-regular solution, which is depended on the Laguerre polynomial of two variables, the Frobenius- Latysheva method is applied. Classification of special curves by means of rank and anti rank, and the basic information about the peculiarities of construction of normal-regular solutions of such systems are presented. The main theorem on the existence of four linearly independent particular solutions, which are expressed in terms of the degenerate hypergeometric function of M.R. Humbert in the form of normally-regular series dependent on Laguerre polynomials of two variables, is proved.

Keywords: Related system, the system of the Laguerre type, the system of Horn, normal-regular solution, special curves, rank, antirank

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The Coinciding Polygonal Numbers and the Related Arithmetical Properties of Tchebyshef Polynomials

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Abstract: We present an algorithm for identification of all coinciding heptagonal and square numbers. The solution to the corresponding Diophantine equation is in the form of three series of ascending solutions. We also specify arithmetic progressions containing certain linear combinations of values of Tchebyshef polynomials at odd triangle numbers.

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Construction of solutions of inhomogeneous systems of Jacobi type

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Abstract: Studying the possibilities of constructing a general solution of a non-homogeneous system of second-order partial differential equations of Jacobi type, closest to ordinary differential equations of the second order. To construct a particular solution of the system is applied the method of undetermined coefficient. Specific examples are established the connection between systems of Jacobi type with one partial differential equation of the second order obtained by adding two equations of the original system.

Keywords: System, particular solution, homogeneous, inhomogeneous, general solution, polynomial, orthogonal

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Assessment of slope stability (on the example of Jalal-Abad in the area of Khamza street)

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Abstract: In this paper, we present the results of an engineering and geological survey to assess the stability of a slope in the area of Khamza street in the city of Jalal-Abad. The climatic characteristics were given from the data of long-term observations at the "Jalalabad" meteorological station. Geological, hydrogeological, geomorphological and tectonic conditions of the terrain are given. The seismicity of the area is estimated by the initial score 9. The physical and mechanical properties of soils have been determined as a result of the laboratory processing of field and laboratory materials, and in accordance with GOST 20522-75 and GOST 25100-95, two engineering-geological elements (IGE) were identified in the studied area: IGE-1: Loess loam, light brown, mostly solid and semi-solid. Density of loamy soils with depth sustained, deviations insignificant. IGE–2: Conglomerate gray, strong, slightly cracked, from the surface is exposed to weathering. The rock density is: $p = 2.2 - 2.4t/m$.

As a result of the research, the following conclusions were obtained:

1. Currently landslide processes of threatening sizes and volumes on the slope are not observed. The calculations of the coefficient of stability allow us to confirm the stable state of the soil mass of the constituent slope for the period of research. With the value of the coefficient of stability Ku¿ 1, the array is fairly stable. 2. Absence of cracks in subsidence, precursors of possible or expected landslide processes, also points to the stability of the slope at a given time. 3. In the case of precipitation of abnormally abundant precipitation and, as a consequence, wetting of the entire mass of loamy strata, especially in the plantar part, to the soft and flowing-plastic state, deformation processes associated with subsidence of loess-like loams in the upper part of the slope where their thickness is greatest are possible.

Keywords: landslide, stability, engineering-geological conditions, coefficient of stability.

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A Discrete-time Holling Type II Model with Allee and Refuge Effects

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Abstract: A discrete-time Holling type II prey-predator model with Allee and the refuge effects is formulated and studied. The existence of the positive equilibrium and Its stability are investigated for both hyperbolic and non hyperbolic cases. Numerical simulations are conducted to demonstrate the theoretical results.

Keywords: Discrete dynamical systems, Holling type

2010 Mathematics Subject Classification: $92B05$

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On a first-order partial differential equation

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Abstract: In this report we study a linear differential equation with first-order partial derivatives, where the coefficients of the equation are given on an unbounded set and has continuous first-order partial derivatives.

$$\frac{\partial u}{\partial x} + \left(\sum_{k=1}^{n} p_{1k}(x) y_k + g_1(x, y_1, \ldots, y_n) \right) \frac{\partial u}{\partial y_1}$$

$$\ldots + \left(\sum_{k=1}^{n} p_{nk}(x) y_k + g_n(x, y_1, \ldots, y_n) \right) \frac{\partial u}{\partial y_n} = 0 \tag{1}$$

where $x \in I \equiv [x_0 \leq x < +\infty), \ x_0 > 0, \ -\infty < y_1, \ldots, y_n < +\infty,$
Theorem 1. If the following conditions are fullfilled:

(1) $p_{ik}(x), i = 1, \ldots, n, \ k = 1, \ldots, n$; are continuous differentiable on I.
(2) $p_{k-1,k-1}(x) - p_{kk}(x) \geq \alpha\varphi(x)$, $x \in I$, $k = 2, \ldots, n. \alpha > 0$, $\varphi(x) \in \mathbb{C}(I)$, $\varphi(x) > 0, \ q(x) = \int\limits_{x_0}^{x} \varphi(s) ds \uparrow +\infty;$
(3) $\lim\limits_{x \to +\infty} \frac{|p_{ik}(x)|}{\varphi(x)} = 0, \ i \neq k, \ i = 1, 2, \ldots, n, \ k = 1, 2, \ldots, n;$
(4) $\lim\limits_{x \to +\infty} \frac{1}{q(x)} \int\limits_{x_0}^{x} p_{kk}(s) ds = \beta_k, \ k = 1, 2, \ldots, n, \ \beta_1 < 0;$
(5) $g(x, y) = colon\, (g_1(x, y_1, \ldots, y_n), \ldots, g_n(x, y_1, \ldots, y_n))$ has a continuous partial derivatives on the set $x_0 \leq x < +\infty, \ x_0 > 0, \ -\infty < y_1, \ldots, y_n < +\infty$, è $g_i(x, 0, \ldots, 0) = 0, \ i = 1, \ldots, n.$ $\|g(x, y)\| \leq \delta(x)\|y\|$, $\lim\limits_{x \to +\infty} \frac{\delta(x)}{\varphi(x)} = 0$, then the equation (1) has a integral basis, which attempts to zero at $x_0 \to +\infty$.

Keywords: equation, first order partial derivatives

2010 Mathematics Subject Classification: 35B35

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Identification hyperbolic problems with nonlocal conditions

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Abstract: In the present study, a identification problem with nonlocal conditions for a one-dimensional hyperbolic equation is investigated. Stability estimates for the solution of the identification problem are established. Furthermore, a first order of accuracy difference scheme for the numerical solution of the identification hyperbolic equations problems with nonlocal conditions is presented. Stability estimates for the solution of the difference scheme are established. Then, this difference scheme is tested on an example and some numerical results are presented.

Keywords: Source identification problem, hyperbolic differential equations, difference schemes.
2010 Mathematics Subject Classification: 35J05, 35J08, 35J25

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Numerical Solution of Nonlocal Elliptic Problems

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Abstract: The well-posedness in various Banach spaces of the local boundary value problem for the elliptic equation in an arbitrary Banach space with the positive operator and its related applications have been investigated by many researchers (see, for example, [1]- [3] and the references given therein).

In the present paper, the second order of approximation two-step difference scheme for the approximate solution of the nonlocal boundary value problem for the elliptic differential equation in an arbitrary Banach space with the positive operator is investigated. The well-posedness of this difference scheme in various Banach spaces is established. In applications, some illustrative numerical results are provided.

Keywords: Well-posedness; coercive stability; positive operators; elliptic equation.

2010 Mathematics Subject Classification: 2010 Mathematics Subject Classification: 35J25, 47E05, 34B27

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Well-posedness of Neumann-type elliptic overdetermined problem with integral condition

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Abstract: In this work, we consider the following Neumann type elliptic overdetermined problem with integral condition to find a function $u \in C^2\left([0,T]\,,\ H\right) \cap C\left([0,T]\,,\ D(A)\right)$ and an element $p \in H$:

$$\begin{cases} -u_{tt}(t) + Au(t) = f(t) + p, \quad 0 < t < T, \\ u_t(0) = \varphi,\ u_t(T) = \int\limits_0^T \alpha\left(\lambda\right) u_\lambda(\lambda) d\lambda + \psi,\ u(\lambda_0) = \zeta. \end{cases}$$

where A is a selfadjoint and positive definite operator in an arbitrary Hilbert space H, smooth function $f(t)$, the elements $\varphi, \zeta, \psi \in D(A)$ and number $\lambda_0 \in (0,T)$ are given. Let the given smooth scalar fuction $\alpha\left(t\right)$ be under condition $\int\limits_0^T \left|\alpha\left(\lambda\right)\right|\ d\lambda < 1.$

In the papers [1–5] well-posedness of various overdetermined elliptic type differential and difference problems are studied.

Abstract results on stability, almost coercive stability and coercive stability estimates for the solution of this problem are established. Later, the abstract results are used to establish well-posedness of overdetermined problem multi-dimensional elliptic equation with integral boundary condition.

Keywords: inverse problem, well-posedness, stability, coercive stability, overdetermination.

2010 Mathematics Subject Classification: 35N25, 35J67

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On the numerical solution of advection-diffusion problems with singular source terms

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Abstract: Partial differential equations with singular source terms are widely used in mathematical modeling of real-life systems in many different fields of science. Singular means that within the spatial domain the source is defined by a Dirac delta function. Solutions of the problems having singular source terms have lack of smoothness, which is generally an obstacle for standard numerical techniques [1–3].

In this work, we consider the initial-boundary value problem with singular source terms

$$(1) \qquad \begin{cases} u_t + au_x = Du_{xx} + k_1\delta(x-\xi_1) + k_2\delta(x-\xi_2), \quad 0 < x < 1, \quad t > 0, \\ u(t,0) = u_L, \quad u(t,1) = u_R, \quad t \geq 0, \\ u(0,x) = \varphi(x), \quad 0 \leq x \leq 1, \end{cases}$$

where $0 < \xi_1 < \xi_2 < 1$ and $\delta(x)$ is a Dirac delta function. We firstly derive the analytical solution of problem (1). Further, we describe the procedure for numerical solution of problem (1) using the standard finite volume method. We provide with numerical illustration for simple test problem.

Keywords: advection-diffusion equation, singular source terms, finite volume method

2010 Mathematics Subject Classification: 65M08, 65M20, 65N08

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A Second Order Difference Scheme for the Numerical Processing of Source Identification Terms in Hyperbolic Equations

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Abstract: In the present study, a second order difference scheme for a one-dimensional hyperbolic equation is presented. Stability estimates for the solution of the difference scheme are established. This difference scheme is tested on an example and some numerical results are presented.
Keywords: Source identification, hyperbolic differential equations, difference schemes, stability.
2010 Mathematics Subject Classification: 35J05, 35J08, 35J25

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Riesz basis property of generalized eigenfunctions for many interval BVTP's with eigenparameter dependent boundary conditions

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Abstract: The main goal of this study is to provide an operator-pencil framework for the investigation of many-interval boundary-value-transmission problems (BVTP) with eigenparameter appearing in the boundary-transmission conditions. By applying an our own approaches the considered problem is transformed into an eigenvalue problem for suitable integral equation in terms of which it is defined a concept of generalized eigenfunctions. We introduce some self-adjoint compact operators in suitable Sobolev spaces such a way that the considered problem can be reduced to an operator-pencil equation. Finally, it is shown that the spectrum is discrete and the set of generalized eigenfunctions form a Riesz basis of the suitable Hilbert space.

Keywords: Boundary value problems, boundary and transmission conditions, eigenvalues, generalized eigenfunctions, Riesz basis.

2010 Mathematics Subject Classification: 34L10, 34L15

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Lower bound estimation for eigenvalues for many interval BVP's with eigenparameter dependent boundary conditions

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Abstract: The present paper deals with multi-interval Sturm-Liouville equations with eigenparameter dependent boundary-transmission conditions. Such type of problems cannot be treated with the usual techniques within the standard framework os classical Sturmian theory. It is well-known that any eigenvalue of the classical Sturm-Liouville problems can be related to its eigenfunction by the Rayleigh quotient and some useful results can be obtained from the Rayleigh quotient without solving the differential equation. For instance, it can be quite useful in estimating the eigenvalues. In this study we present a new technique for investigation some computational aspects of the eigenvalues. Particularly, we give an operator-pencil formulation of the problem and establish lower bound estimation for eigenvalues by using modified Rayleigh quotient.

Keywords: Boundary value problems, boundary and transmission conditions, eigenvalues, generalized eigenfunctions, Riesz basis.

2010 Mathematics Subject Classification: 34L10, 34L15

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Source Identification Problems for Schrödinger Differential and Difference Equations

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Abstract: In the present study, a source identification problem for a one-dimensional Schrodinger equation is investigated. Stability estimates for the solution of the source identification problem are established. A first order of accuracy difference scheme for the numerical solution of this problem is presented. Stability estimates for the solution of this difference scheme are established. Numerical results are provided.

Keywords: Source identification problem, Schrodinger differential equation, difference scheme.

2010 Mathematics Subject Classification: 35Q41, 65M06, 34G10

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Stability of the third order partial differential equations with time delay

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Abstract: In the present paper, the initial value problem for the third order partial differential equations with time delay in a Hilbert space with self-adjoint positive definite operator is investigated. The main theorem on stability of this problem is established. The application of this theorem is presented. Additionally, illustrative numerical results are provided.

Keywords: Partial differential equation, stability, time delay, difference scheme

2010 Mathematics Subject Classification: 35G10, 65M12, 34D20

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Numerical solutions of the system of partial differential equations for observing epidemic models

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Abstract: In the present paper, stability of initial-boundary value problem for the system of partial differential equations for observing HIV mother to child transmission epidemic models is studied. Applying operator approach, theorems on stability of this problem and of difference schemes for approximate solutions of this problem are established. The generality of the approach considered in this paper, however, allows for treating a wider class of multidimensional problems. Numerical results are provided.

Keywords: Difference schemes, system of partial differential equation, epidemic models, realization in computer.

2010 Mathematics Subject Classification: 35K40, 65M12, 92B05

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A Mathematical Cancer Model with BCG Immunotherapy Combined with Immune Checkpoints Inhibitors: an Optimal Control Approach

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Abstract: We present a mathematical model of cancer growth in the bladder that includes the immune cells, BCG, immune checkpoints and drug therapy (checkpoint inhibitor) in the form of a control function. The control function blocks the action of immune checkpoints on the immune system. Our aim here is to apply optimal control theory to find a control strategy that will minimize the number of cancer cells in the bladder and cost of control. Existence of the optimal control is stated and Pontryagin's maximum principle is used to characterize the nature of the control function. The optimality system obtained gives a two-point boundary value problem; hence, we use the forward-backward sweep method to present the numerical solutions of the system. The optimality conditions and characterization of the control are discussed.

Keywords: Optimal control, mathematical model, Hamiltonian, cancer, immune checkpoint, Pontryagin's principle, forward-backward sweep method.

2010 Mathematics Subject Classification: 35J05, 35J08, 35J25

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An Interval Based Social Accounting Matrix for a Regional Economy

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Abstract The assessment of the major impact of economic events is based primarily on the input-output model. In all of the existing input-output models exact information is used. Yet, in real life, we can never usually find such exact information for problems in particular related to the future. An expansion of the input-output model is the Social Accounting Matrix (SAM) and the SAM builds on Leontief's input-output tables. In this paper we construct a SAM model for a regional economy and initial information is described by interval numbers. For the given region Northern Cyprus, where a scarcity of data exists, interval information based SAM, based on the best available information is suggested.

Keywords: Input-output model, Social Accounting Matrix, interval number, regional economy

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An Input-Output Analysis for a Regional Economy: The Case of North Cyprus

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Abstract Input-output analysis is an economic tool that is used in order to measure the impact of an existing, planned or expected business operation, decision or event on the economy. This paper presents an Input-output Analysis with the latest available information for a national economy, which is an important source of information to understand the inter-relations existing among the different sectors of an economy. This paper focuses on the North Cyprus economy and the input-output analysis was used to measure the impact of changes in two sectors, namely Tourism and Education on the national economy.

Keywords: Input-output analysis, input-output table, Education, Tourism, North Cyprus

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Source Identification Problems for Two Dimensional Neutron Transport Differential and Difference Equations

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Abstract: In this study, an inverse problem concerning two dimensional neutron transport equation with a time-dependent source control parameter is considered. For the approximate solution of this problem a first order of accuracy difference scheme is constructed. Finite difference schemes are presented for identifying the control parameter. Stability inequalities for the solution of this problem are established. The results of a numerical experiment are presented, and the accuracy for this inverse problem is discussed.

Keywords: Source identification problem, Two dimensional neutron transport equation, difference scheme.

2010 Mathematics Subject Classification: 65M12, 65M06, 82D75

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New type Sturm-Liouville eigenvalue problem with interior singular point

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Abstract: This work investigates a kind of discontinuous Sturm-Liouville problem with boundary conditions, and transmission conditions are imposed at an singular interior point. We establish some spectral properties of the considered problem. At first, we shall define some new Hilbert spaces and give some inequalities which is needed for further investigation. Then, we suggest some methods for solving these Sturm-Liouville problems with transmission conditions.

Keywords: Sturm-Liouville problems, eigenvalues, transmission conditions.

2010 Mathematics Subject Classification: 34L10, 34L15

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New least-squares adjustments of nuclear binding energies of atomic nuclei

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Abstract A new least-squares adjustment of binding energies of atomic nuclei based on the last updated atomic mass evaluation AME2016 has been considered. The new set of parameters of mass formula, obtained in this work, may reduce the relative error in binding energies when calculated in the framework of mass formula. A special treatment has been made for medium and heavy nuclei with atomic masses greater than 50.

Keywords: Mass formula, least-squares adjustments, AME2016, binding energy

Adaptive conditional integrators sliding mode control scheme for a class of nonlinear systems

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Abstract: The present paper is dedicated to the presentation and implementation of conditional integrators sliding mode controllers based on adaptive approach in order to synthesise tracking errors and to ensure Lyapunov stability, of all system nonlinearities and desired tracking trajectories. The adaptive approach allows an on-line estimation of nonlinear system parameters to use them in conditional integrator sliding mode control scheme. Indeed the proposed control law needs the exact system model to give good performances. The complexity of the nonlinear systems makes it hard to know its parameters. However, the integrator of sliding mode controllers is introduced in such a way that it provides integral action only (conditionally), effectively eliminating the performance degradation.

Finally, the proposed controller design is illustrated by application to control single inverted pendulum with simulation in MATLAB environment.

Keywords: Nonlinear system, Sliding mode, Adaptive approche, Stability, Conditionnal integrators, Invert pundulum.

2010 Mathematics Subject Classification: 34H15, 93C10, 93C40

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Review of modeling and simulation of natural nuclear reactors

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Abstract: The present work intend to share the up-to-date modeling and simulation performed to explain the criticality occurrence in natural nuclear systems. Such systems were ignited 2 billion years ago, in the Oklo Uranium deposit, in Gabon (West Africa). The studied system is modelled by taking into account the geological context. The simulations were carried out with a Monte-carlo based code, known as MCNP; widely used for neutron physics and nuclear reactors calculation. By looking for the criticality conditions we resolved the following equation :

$$k_{eff}(V_{UO2}, \phi_{TOT}, e, R, t_{B10}) = 1 \tag{2}$$

with: V_{UO2}: volume fraction of uraninite, ϕ_{TOT}: totally saturated ore porosity, e, R: geometrical dimensions, t_{B10}: Boron-10 equivalent content.

Numerical simulations carried out with MCNP, generated a set of critical solutions as a parametrical curves, called Isocritical lines. Each point from such a line represent a possible natural critical configuration, given in correpsonding conditions of Oklo case. An isocritical line represent an interface in space parameters, separating a barren space (subcritical system) from fissile space (critical and supercritical systems). Although, any evolving natural nuclear reactor should at least reach the criticality line to start a dynamic which lead it to a new geological and physical state: Fuel burn-up, mineral alteration of the container ore, transmutation, fission products accumulation, etc...

Detailed techniques used to model and simulate such systems are given in previous works [1].

Keywords: Oklo, Modeling and Simulation, MCNP, Monte-Carlo, Uranium, Natural Reactors

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Delay Epidemic Model with and without Vaccine

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Abstract: In this work, two models with and without vaccine are constucted. For both model basic reproduction numbers are founded as a

$$R_0^1 = \frac{e^{-\mu\tau}\beta\Lambda}{\gamma+\mu+d}, R_0^2 = \frac{e^{-\mu\tau}\Lambda}{(\gamma+\mu+d)(\mu+r)}\left(\beta + \frac{kr}{\mu+1-k}\right).$$

From the basic reproduction ratios it can be seen that when there is no vacccine, disease can only controled with reducing the infectious rate β or decreasing the incubation period. However, when there is a vaccine it is enough to increase the rate of the vaccine r to controlled the disease. In addition to this if transmission rate of infectious also decrease then the disease will disappear faster.

Delay effect on this model is very rare. When we are talking about delay on this paper we mean that incubation period. If we have enough vaccine the effect of delay is very tiny. However if there is no vaccine you can see the effect of delay.

Two equilibria which are disease free and endemic equilibriums are found and with using Lyapunov function it is shown that the global stabilities of each equilibria for both model. For the first model it is found that DFE E_0 is globally asymtotically stable when $R_0^1 < 1$ and endemic equilibrium E_1 is always asymtotically stable. With using similar method it is shown that E_0 is asymtotically stable when $R_0^2 < 1$ and E_1 is always global asymtotically stable for model 2. In last section numerical simulations are given for both model.

Keywords: Delay, Epidemic, Modelling, Vaccine differential equation.

2010 Mathematics Subject Classification: 35N05, 37C75, 34K50

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Principle of Independence of Continuation of Functions k- Significant Logic From Coding

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Abstract: In this paper we consider not everywhere defined functions of k-valued logic. The problem of logical continuation of functions of k-valued logic in the class of disjunctive normal forms is investigated. We prove a theorem on the invariant extension of functions of k-valued logic in the class of disjunctive norms, which does not depend on the accepted coding. An algorithm is constructed for constructing a set of invariant points that are independent of the received encoding.

Keywords: Encoding, logic, conjunction, disjunction, sets, quasi-Boolean invariant, synthesis, equivalent

We consider the function $F\left(x_1,\, x_2,\, \ldots,\, x_n\right)$ k - valued logic, given on $M \subseteq E_k^n$:

$$F(\tilde{x}) = \gamma_j, \text{ if } \tilde{x} \in M_j, \left(j = \overline{0,\, k-1}\right),$$

where $M = \bigcup\limits_{i=0}^{m} M_i$ and $M_i \cap M_j = \emptyset$ for $i \neq j$. We denote by $\{\pi\}$ the set of all permutations of the set $\{0, 1, \ldots, k-1\} : \pi = \left(i_0,\, i_1, \ldots,\, i_{k-1}\right)$. The functions $F_\pi\left(\tilde{x}\right) = i_j$, if $\tilde{x} \in M_j$, $\left(j = \overline{0,\ k-1}\right)$, we call π is a permutation of $F\left(\tilde{x}\right)$. We will say that a point $\tilde{\alpha} \in E_k^n \backslash M$ preserves the encoding (code) of a set $M_j \subseteq M$, $j = \overline{0,\ k-1}$ with respect to a permutation pi , if $\mathfrak{M}_{\Sigma TF}\left(\tilde{\alpha}\right) = j$ and $\mathfrak{M}_{\Sigma TF_\pi}\left(\tilde{\alpha}\right) = i_j$. A point $\tilde{\alpha} \in E_k^n \backslash M$ is called a point that preserves the encoding of the set M_j, if it preserves the encoding M_j with respect to any permutation $\pi \in \{\pi\}$. Let $\mathfrak{M} = \mathop{\mathrm{V}}\limits_{\pi:\pi\in(\pi)} \mathfrak{M}_{\Sigma TF_\pi}$.

Theorem. The point $\tilde{\alpha} \in E_k^n \backslash \bigcup\limits_{i=0}^{k-1} M_i$ preserves the code of the set M_j, then and only if, when:

1) in DNF $\mathfrak{M}$ there is an EK $\mathfrak{A}$ such that $N_{\mathfrak{A}} \cap M_j \neq \emptyset$, $\tilde{\alpha} \in N_{\mathfrak{A}}$, $N_{\mathfrak{A}} \cap M_i = \emptyset$, where $(i = 0,\, \ldots, j-1,\, j+1,\, \ldots,\, k-1)$;

2) each interval $N_{\mathfrak{A}}$ where EK $\mathfrak{A}$ belongs to the set $\mathfrak{M}$ intersects with M_i, $(i \neq j)$ and contains the point M_j.

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Difference schemes for delay Schrödinger equation

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Abstract: In the present paper, the first and second order of accuracy stable difference schemes for the Schrödinger equation with time delay in a Hilbert space are presented [1]. Theorems on stability estimates for the solution of these difference schemes are established by using approaches of [2]. The application of theorems on stability of the approximate solution of difference schemes for the three types of delay Schrödinger partial differential problems is provided. Additionally, some illustrative numerical results are given.

Keywords: Schrödinger differential equations, difference schemes, stability estimates

2010 Mathematics Subject Classification: 65M06, 35R11

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On numerical solution of the Schrödinger-parabolic equations

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Abstract: The nonlocal boundary value problem for Schrödinger-parabolic equation is considered. The stability estimates for the solution of the given problem is established. The first and second order of difference schemes are presented for approximately solving a specific nonlocal boundary problem. The theoretical statements for the solution of these difference schemes are supported by the result of numerical examples.

Keywords: Nonlocal boundary value problem, Difference scheme, Stability.

2010 Mathematics Subject Classification: 34B10, 65L12, 35B35

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On the numerical solution of nonlinear system of coupled sine-Gordon equations

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Abstract: In this paper, a system of nonlinear coupled sine-Gordon, which have some powerful applications in physics and biology is considered. A special case of this system, which describe the open states in DNA double helices is studied. Numerical solution of this system is obtained by finite difference method with fixed point iteration. Some examples are considered and the results of numerical experiments are presented.

Keywords: nonlinear boundary value problems, difference equations, numerical analysis

2010 Mathematics Subject Classification: 34B15, 39A10, 97N40

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A note on stable difference schemes for a third order partial differential equation

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Abstract: Boundary value problems for third order partial differential equations have been a major research area in thermal conductivity [1], micro-scale heat transfer [2] and modern physics. The well-posedness of various nonlocal and local boundary value problems for partial differential and difference equations have been studied extensively by many researchers (see [1, 1–4, 6–8] and the references given therein). In this study, the nonlocal boundary value problem for the third order partial differential equation with a self-adjoint positive definite operator in a Hilbert space is studied. The stability estimates for the solution of the problem are established. In applications, the stability estimates for the solution of two types of third order partial differential equations are obtained. A stable three-step difference schemes for the approximate solution of the problem is presented. The theorem on stability of these difference schemes is established. In applications, the stability estimates for the solution of difference schemes of the approximate solution of nonlocal boundary value problem for third order partial differential equations are obtained. For the numerical analysis, a first and a second order of approximation difference schemes for a one dimensional third order partial differential equation are presented. Moreover, some numerical results are given.

Keywords: Nonlocal boundary value problems, Stability, Difference scheme,Third order partial differential equation, Self-adjoint positive definite operator, Hilbert space.

2010 Mathematics Subject Classification: 35G15; 47A62

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Invariant difference schemes for sine-Gordon equations

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Abstract: In this paper, we consider invariant differential and present corresponding invariant difference equations which preserve the Lie group symmetries of the sine-Gordon equation. We present invariant difference schmes for the approximate solution of sine-Gordon equation which conserve uniformity and orthogonality of meshes.

have some powerful applications in physics and biology. A special case of this system, which describe the open states in DNA double helices is studied. Numerical solution of this system is obtained by finite difference method with fixed point iteration. Some examples are considered and the results of numerical experiments are presented.

Keywords: nonlinear boundary value problems, difference equations, numerical analysis

2010 Mathematics Subject Classification: 34B15, 39A10, 97N40

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Unification of $q-$exponential function and related $q-$numbers and polynomials

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Abstract: The main purpose of this paper is to introduce and investigate a class of generalized Bernoulli polynomials and Euler polynomials based on the generating function. we unify all forms of $q-$exponential functions by one more parameter. we study some conditions on this parameter to related this to some classical results for q-Bernoulli numbers and polynomials. This unification reach to the nonlinear system of equations for that extra parameter and give us the general form of symmetric q-exponential function.

Keywords: q-Exponential function, q-calculus, q-Bernoulli polynomials, q-Euler number.

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Suitable variations for degenerate modes of the simplex method

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The problem of finding a nontrivial solution of problem

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n \le 0, \\ \ldots \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n \le 0 \end{cases} \tag{1}$$

at observance of restrictions

$$x_i \ge 0, i = \overline{1, n}. \tag{2}$$

The question of existence of the untrivial solution of this task is key in case of the degenerate modes at the solution of tasks of the linear programming (LP) by simplex method [1]. It is shown in work [2].

Here we will specify an effective method of finding of the solution of an objective if it exists.

Let $A_j = (a_{1j}, \ldots, a_{mj})^T$ be normalized vector. Let us

$$\|A\| = \sqrt{\sum_{i=1}^{m} a_{ij}^2} = 1 \tag{3}$$

It does not reduce a community of consideration of the considered task [2]. Having entered in addition m of variables $x_i \ge 0, i = \overline{n+1, n+m}$ we will transform inequalities (1) to system of the linear equations

$$\begin{cases} a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n + x_{n+1} = 0, \\ \ldots \\ a_{m1}x_1 + a_{m2}x_2 + \ldots + a_{mn}x_n \ + x_{n+m} = 0. \end{cases} \tag{4}$$

The general restriction for all variables now have an appearance

$$x_i \ge 0, \quad i = \overline{1, n+m} \tag{5}$$

Denote by A_{n+k} for any $k = \overline{1, m}$ vector column $m \times 1$ with zero elements in each row expect k^{th} row whode element is one. Then, we can write system (4) in a vector form

$$\sum_{j=1}^{n+m} x_j A_j = 0 \tag{6}$$

We form a set of U in m - measured Euclidean space of R^m with elements $u = (u_1, u_2, \ldots, u_m)^T$ such that

$$u_k = a_{11}x_1 + a_{12}x_2 + \ldots + a_{1n}x_n + x_{n+k}, k = \overline{1, m} \tag{7}$$

representing any linear combinations of vectors A_j, $j = \overline{(1, n+m)}$, with the non-negative coefficients of x_j satisfy the condition

$$\sum_{i=1}^{n+m} x_i = 1 \tag{8}$$

We will note that the system (4) is uniform; therefore the condition (8) doesn't reduce a community of consideration of a task [2].

Geometrically set of U represents a polyhedron [1] in R^m space, with vertices of A_j, $j = \overline{(1, n+m)}$, located on the sphere S with single radiuses. It is known that any point of u limited polyhedron in R^m is representable in the form of a linear combination but at most of $m+1$ â“ its vertices

$$u = \sum_{j=1}^{n+m} x_j A_j, \tag{9}$$

at the same time restrictions (5) and (8) are observed. Thus, if

$$0 \in U, \tag{10}$$

that representation takes place $0 = \sum_{j=1}^{n+m} x_j A_j$, so the conditions (5), (6) and (8) are satisfied.

Theorem: In order that the initial task (1),(2) had the decision enough performance of inclusion (10).

This theorem has allowed to make effective algorithm and the program of the solution of LP tasks with the degenerate modes [2]. Made a comparison of the proposed method with known modifications of the simplex method [2].

2010 Mathematics Subject Classification: 35J05, 35J08, 35J25

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Numerical solutions of source identification problem for hyperbolic-parabolic equations

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Abstract: Numerical methods for partial differential equations with unknown source terms have been studied recently by many researchers (see for instance [1–3] and the references therein).

In this work, a numerical study is presented of boundary value problem for hyperbolic-parabolic equations

$$(1) \qquad \begin{cases} u_{tt}(t,x) - \big(a(x)u_x(t,x)\big)_x = p(x) + f(t,x), \quad 0 < x < 1,\ 0 < t < 1, \\ u_t(t,x) - \big(a(x)u_x(t,x)\big)_x = p(x) + g(t,x), \quad 0 < x < 1,\ -1 < t < 0, \\ u(0+,x) = u(0-,x),\ u_t(0+,x) = u_t(0-,x), \quad 0 \le x \le 1, \\ u(-1,x) = \varphi(x),\ u(1,x) = \psi(x), \quad 0 \le x \le 1, \\ u(t,0) = u(t,1) = 0, \quad -1 \le t \le 1. \end{cases}$$

where $p(x)$ is an unknown source term. The first and second order of accuracy difference schemes for the approximate solution of problem (1) are constructed and the numerical procedure for implementation of these schemes is discussed. These difference schemes are implemented for a simple test problem and the numerical results are presented.

Keywords: Difference scheme, source identification problem, hyperbolic-parabolic equation

2010 Mathematics Subject Classification: 65M06, 65N05, 35M13, 35R30

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On the stable difference scheme for the time delay telegraph equation

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Abstract: In this study, the first order of accuracay difference scheme

$$\begin{cases} \frac{u_{k+1}-2u_k+u_{k-1}}{\tau^2}+\alpha\frac{u_{k+1}-u_k}{\tau}+Au_{k+1}=aAu_{\left[\frac{k}{N+1}\right]N}, \\ 1\leq k<\infty,\ N\tau=1,(m-1)N\leq k\leq mN, m=1,2,... \\ u_0=\varphi, \quad ((1+\alpha\tau)I+\tau^2 A)\frac{u_1-u_0}{\tau}=\psi \end{cases}$$

is considered for the approximate solution of the initial value problem for the telegraph equation with time delay

$$\begin{cases} \frac{d^2u(t)}{dt^2}+\alpha\frac{du(t)}{dt}+Au(t)=aAu([t]),\ 0<t<\infty, \\ u(0)=\varphi, \quad u'(0)=\psi \end{cases}$$

in a Hilbert space H with a self-adjoint positive definite operator A, $A>\delta I$. Here φ and ψ are elements of $D(A)$ and $[t]$ denotes the greatest-integer function and $\delta>\frac{\alpha^2}{4}$. The stability of this difference scheme is obtained and numerical results for a test problem are presented.

Keywords: Telegraph equation; time delay; difference schemes; stability.

2010 Mathematics Subject Classification: 35L10**,** 65N06.

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Numerical solutions of source identification problem for hyperbolic-parabolic equations

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Abstract: Numerical methods for partial differential equations with unknown source terms have been studied recently by many researchers (see for instance [1–3] and the references therein).

In this work, a numerical study is presented of boundary value problem for hyperbolic-parabolic equations

$$(1)\qquad \begin{cases} u_{tt}(t,x) - \big(a(x)u_x(t,x)\big)_x = p(x) + f(t,x), \quad 0 < x < 1,\ 0 < t < 1, \\ u_t(t,x) - \big(a(x)u_x(t,x)\big)_x = p(x) + g(t,x), \quad 0 < x < 1,\ -1 < t < 0, \\ u(0+,x) = u(0-,x),\ u_t(0+,x) = u_t(0-,x), \quad 0 \le x \le 1, \\ u(-1,x) = \varphi(x),\ u(1,x) = \psi(x), \quad 0 \le x \le 1, \\ u(t,0) = u(t,1) = 0, \quad -1 \le t \le 1. \end{cases}$$

where $p(x)$ is an unknown source term. The first and second order of accuracy difference schemes for the approximate solution of problem (1) are constructed and the numerical procedure for implementation of these schemes is discussed. These difference schemes are implemented for a simple test problem and the numerical results are presented.

Keywords: Difference scheme, source identification problem, hyperbolic-parabolic equation

2010 Mathematics Subject Classification: 65M06, 65N05, 35M13, 35R30

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On the stable difference scheme for the nonlocal fractional PDE

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Abstract: In the present study, we consider the nonlocal boundary value problem for fractional parabolic equations. Firstly, we construct the first order of accuracy difference scheme for solution of nonlocal boundary value problem using the self-adjoint positive defined operator A. Then, the stability estimates for the solution of these difference scheme and some theoretical results are given.

Keywords: Nonlocal BVP; fractional PDE; difference schemes; stability.

2010 Mathematics Subject Classification: 35R11, 34K37.

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New phenomenas for degenerate nonlinear cross system with convective transfer and absorption

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Abstract: The purpose of this article is to investigate properties of the process of a nonlinear diffusion-reaction with heterogeneous density in the domain $Q = \left\{(t,x) : t > 0, x \in R^N\right\}$:

$$\begin{aligned} \frac{\partial u}{\partial t} &= div\left(v^{m_1-1}\left|\nabla u^k\right|^{p-2}\nabla u\right) - div(c(t)u) - \gamma_1(t)u^{\beta_1}, \\ \frac{\partial v}{\partial t} &= div\left(u^{m_2-1}\left|\nabla v^k\right|^{p-2}\nabla v\right) - div(c(t)v) - \gamma_2(t)v^{\beta_2}, \end{aligned} \tag{1}$$

$$u\left(0,x\right) = u_0\left(x\right) \geq 0, v\left(0,x\right) = v_0\left(x\right) \geq 0,\ x \in R^N \tag{2}$$

where $k \geq 1$, p, m_i , β_i, $i = 1,2$ - given positive numbers, $\nabla(.) - \underset{x}{grad}(.)$, functions $u_0\left(x\right),\ \ v_0\left(x\right) \geq 0,\ x \in R^N, 0 < \gamma_i(\mathrm{t}) \in C(0,\infty),\ i = 1,2.$

The system (1) describes a set of physical processes, for example process of mutual reaction - diffusions, heat conductivity, a polytropical filtration of a liquid and gas in the nonlinear environment whose capacity equal to $\gamma_1(t)u^{\beta_1}$, $\gamma_2(t)v^{\beta_2}$. Particular cases $(k = 1,\ \ p = 2)$ of the system were considered in works (see for instance [1-3]).

In this work is established the estimate of a weak solution, the Fujita type critical exponent, a global solvability of solution, the phenomena of a finite speed of perturbation of disturbance, space localization, localized a wave type structure.

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Mathematics Education

Problems of heat conduction with different boundary conditions in noncylindrical domains

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Abstract: In a degenerating domain $G = \{(x;\, t):\quad t > 0, \quad 0 < x < t\}$ homogeneous problems for the heat equation with different boundary conditions are considered.

Problems with conditions one of which contains the time derivative are reduced to singular integral equations with incompressible kernel:

$$\varphi(t) - \int\limits_0^t K(t,\, \tau)\, \varphi(\tau)\, d\tau = 0, \tag{1}$$

where

$$K(t,\, \tau) = \frac{1}{2a\sqrt{\pi}} \left\{ -\frac{t+\tau}{(t-\tau)^{\frac{3}{2}}} \exp\left(-\frac{(t+\tau)^2}{4a^2(t-\tau)} \right) + \frac{1}{(t-\tau)^{\frac{1}{2}}} \exp\left(-\frac{t-\tau}{4a^2} \right) \right\}.$$

It is shown that the norm of the integral operator in equation (1), acting in classes of continuous functions, is equal to 1.

The integral equation for other BVP is considered in [1]. Its solution was found in an explicit form. It is shown that the original homogeneous boundary value problems have nontrivial solutions in certain functional classes.

The research and writing of this work was carried out a grant from the Ministry of Science and Education of the Republic of Kazakhstan (Grant No. AP05132262).

Keywords: Volterra integral equation of the second kind, incompressibility of the kernel, degenerating domain

2010 Mathematics Subject Classification: 35D05, 35K20, 45C05, 45D05

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Problems of heat conduction in noncylindrical domains with special boundary conditions containing the time derivative

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Abstract: The paper researches pseudo - Volterra integral equations that arise in the study of the boundary value problem for the equation of heat conduction in a degenerating angular domain

(1) $$\frac{\partial u(x,t)}{\partial t} - a^2 \frac{\partial^2 u(x,t)}{\partial x^2} = g(x,t),\ \{x,t\} \in G = \{x,t:\ -t < x < t,\ t > 0\};$$

(2) $$-\left.\frac{\partial u(x,t)}{\partial x}\right|_{x=-t} + \frac{d\tilde{u}_1(t)}{dt} = h_1(t),\ \left.\frac{\partial u(x,t)}{\partial x}\right|_{x=t} + \frac{d\tilde{u}_2(t)}{dt} = h_2(t),$$

where $\tilde{u}_1(t) = u(-t,t),\ \tilde{u}_2(t) = u(t,t)$. [1]

The peculiarity of problem (1) – (2) lies in the fact that both boundaries are mobile and the boundary conditions contain derivatives with respect to time. It is shown that the corresponding homogeneous integral equations in the class of essentially bounded functions have nonzero solutions.

The research and writing of this work was carried out a grant from the Ministry of Science and Education of the Republic of Kazakhstan (Grant No. AP05132262).

Keywords: pseudo - Volterra integral equation, equation of heat conduction, angular domain

2010 Mathematics Subject Classification: 35D05, 35K20, 45C05, 45D05

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The uniqueness of the solution of the two-dimensional direct problem of a wave process with an instantaneous source and a flat boundar

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Abstract: This article is devoted to the justification of the uniqueness of the solution of the problem. A two-dimensional direct generalized problem of hyperbolic type is reduced to a regular problem with data on characteristics and the uniqueness of the solution of this problem is shown.

We consider the following problem:

$$u''_{tt}(z,y,t)+\frac{b_1(z,y)}{c_1(z,y)}u'_t(z,y,t)=\frac{\sigma_1(z,y)}{c_1(z,y)}\Delta u(z,y,t)+\frac{\sigma'_{1z}(z,y)}{c_1(z,y)}u'_z(z,y,t)+$$

$$+\frac{\sigma'_{1y}(z,y)}{c_1(z,y)}u'_y(z,y,t)+\frac{a_1(z,y)}{c_1(z,y)}u(z,y,t),\quad (z,t)\in R^2_+,\quad y\in R, \tag{1}$$

$$u(z,y,t)|_{t<0}\equiv 0,\quad u'_z(z,y,t)|_{z=0}=-\frac{1}{2}(r(y)\delta(t)+h(y)\theta(t)), \tag{2}$$

where $a_1(z,y), b_1(z,y), c_1(z,y), \sigma_1(z,y)$ are the coefficients of the equation describing the physical parameters of the medium, $u(z,y,t)$ is the perturbation of the medium, $\Delta u(z,y,t)=u''_{zz}+u''_{yy}$ is the Laplace's operator, $r(y), h(y)$ are source functions, $\delta(t)$ is the Dirac's delta-function, $\theta(t)$ is the Heaviside's theta-function.

Keywords: two-dimensional, direct problem, wave processes, source, instantaneous, flat boundary, uniqueness of the solution

2010 Mathematics Subject Classification: 45K05, 35L10

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Numerical solution of a two-dimension direct problem of the wave process

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Abstract: The problem of propagation of a potential along a nerve fiber, which is described by a telegraph equation of parabolic type, is considered.This problem is reduced to the inverse problem of the hyperbolic equation and the last problem is solved by a numerical finite-difference method, using methods:rectification of characteristics, isolation of features.A theorem on the convergence of an approximate solution to the exact solution of the inverse problem is proved.In connection with the equivalence of the problem, an approximate calculation of the membrane capacity of the problem is determined. Consider the following problem:

$$\begin{gathered}\frac{\partial^2 u(z,y,t)}{\partial t^2} + \frac{b_1(z,y)}{c_1(z,y)}\frac{\partial u(z,y,t)}{\partial t} = \frac{\sigma_1(z,y)}{c_1(z,y)}\triangle u(z,y,t) \\ + \frac{\sigma_{1z}(z,y)}{c_1(z,y)}\frac{\partial u(z,y,t)}{\partial z} + \frac{\sigma_{1y}(z,y)}{c_1(z,y)}\frac{\partial u(z,y,t)}{\partial y} + \frac{a_1(z,y)}{c_1(z,y)}u(z,y,t), \\ (z,t) \in R_+^2, \quad y \in R, \end{gathered} \tag{1}$$

$$u(z,y,t)|_{t<0} = 0, \qquad \left.\frac{\partial u(z,y,t)}{\partial z}\right|_{z=0} = -\frac{1}{2}(r(y)\delta(t) + h(y)\theta(t)) \tag{2}$$

Where a_1, b_1, c_1, σ_1 - coefficients of the equation, meaning some physical parameters of the environment, $u(z,y,t)$ - Environment pressure, $r(y), h(y)$ - functions of environment sources, $\delta(t), \theta(t)$ - Dirac delta function and theta function Heaviside, $R_+ = \{x \in R : x \geq 0\}$. The direct problem is to determine the pressure of the medium $u(z,y,t)$ for known functions of the coefficients of the equation and the function of the sources. In this paper, all these tasks are grouped, systematized, summarized results, and numerical solutions of the problem.

Keywords: Two-dimensional, direct problem, wave processes, instantaneous source, cord, numerical solution

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If I Could Write Mathematics It Would be a Novel: A new Approach; Curiosity and What ⇔How⇔Why

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Abstract: The beauty of mathematics and geometry can be discovered by young people and children through stories. In this project, we opened a new window in the field of actual mathematical articles with an interesting, remarkable and curious writing. In this writing, we have tried to convey misunderstandings and/or misconceptions of mathematics to many people from seven to seventy years of age in a fun, enjoyable and coherent way. We aimed to bring readers together with "If I Could Write Mathematics, It Would be a Novel", which will consist of mathematical stories within the nature of the basic concepts and proofs of geometry and mathematics.

In order to start with a sample story in this corner, we shared my daughter's project that she had done in fifth grade with contribution of a new approach. In her project, she aimed to describe the number of π, one of the most mysterious numbers in mathematics, in a story style to her classmates and to many people from seven to seventy years of age who could understand without feeling bored or annoyed by making it fun, enjoyable, easy and clear with a different, original method. Moreover, a new approach has been presented with hints and the questions "What⇔How⇔Why" in this story to actual mathematics story books that have not been made so far. Thus, we have demonstrated in a fun way how to learn many concepts, methods of proof and geometric drawing with a new and curious approach; hints, noticeable warnings, remarkable questions and signs in different colors.

Keywords: Mathematics education, proof methods, mathematical reasoning

2010 Mathematics Subject Classification: 97A99, 97E50, 03F03, 97E50

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The Misuse of Mathematical Proof Methods: New Approaches and Suggestions

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Abstract: In this research, some general misuses of mathematical proof methods and interchangeably used concepts in Turkish sources, which do not correspond conceptually, were analyzed. It was found that there is not a common language in mathematical proof methods and there are inconsistencies or missing topic descriptions in Turkish literature (math textbooks, lecture books, articles etc.). The importance and necessity of mathematical proof methods was expressed and some nice general suggestions were given in some Turkish articles. However, in these articles, it has not been seen that remarkable, specific and new recommendations or methods were suggested for a better understanding of the mathematical proof methods and awareness of the misuse of proof methods for Turkish students.

A survey attempting to determine how well the proof methods are known and to what extent students are aware of the misuse of proof methods was conducted. Survey questions including Proof Opinion Questionnaire, Mathematical Proof Knowledge Test were given to students in primary mathematics education and mathematics departments. Interviews were arranged with volunteer students. Questions such as; giving reasons why students picked that answer, which proof methods were used, why that method was chosen, how it can be solved with another method, whether there are similar methods to the used methods were examined. Some questions preferred to evaluate whether they know the concepts of mathematical proof methods properly or not. It is concluded that teacher canditates and academician canditates do not know mathematical proof methods properly, they either confuse the methods or misuse them. Especially, this can be seen in 2015- 2016 academic year eleventh grade advanced math textbooks, where there were mistakes and inconsistencies in mathematical proof methods. It was a gratifying development that some of these books were generally revised after 2016. In order to minimize the mistakes and inconsistencies and to understand better the mathematical proof methods, different approaches having not been made so far will be suggested to the mathematics teachers, university academicians and ministry of education.

Keywords: Mathematics education, proof methods, mathematical reasoning

2010 Mathematics Subject Classification: 97A99, 97E50, 03F03, 97E50

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A different critical approach to the teaching and obtaining of some elementary geometric inequalities

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Abstract: In this research, some misuses of an elementary geometric inequality used concepts in Turkish sources were analyzed. It was found that there are inconsistencies or incorrect results for this elementary geometric inequality in Turkish resourses (university preparation course books and math textbooks, etc.). A more useful inequality by specifying the largest lower and smallest upper bounds was found, to be able to end the inaccuracy of given inequality. After searching this geometric inequality in Turkish geometry studies, it was observed that a researcher had studied in [4,5] such as this work. But, in this study, our proofs are completely different, more simple and clearer. In [5], he gave a problem related to this inequality. He insisted this problem he had given could not be solved through geometry but through calculus. In the second part of this study, solution to this problem using the geometrical way was given successfully. Lastly, a new different approach for teaching these elementary geometric inequalies would be suggested for for the first time for the mathematics teachers and writers of math books, since the erroneous questions about this geometric inequality have been prepared and solved or answered incorrectly in the most widely used resource books by students such as those given in [6-10] until this time.

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Abstract: A new approach for ranking visual career counselling based on distance measures is explained. In this paper, the traditional method of career counselling will be carried out by means of distance consulting services to make career counselling. Especially, professional education in the capacity limitation and professional working demand for education has led to a proliferation of virtual applications. Hence, e-Visual Career Application has been developed as a web based program for clients who have taken professional counselling efficiently. With this application, it will help clients in the process of discovering and making decisions to their interests, abilities, personalities, skills and values. This paper relies on the implications of introducing online visual career consulting in the login sessions and how it affects the people (supervisor and client), the processes (consulting), and the organizations involved.

Keywords: Visual Career Counselling, Traditional Method, Virtual Application, e-Visual Career Application**References:**

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Mathematics through the Mathematical Modeling Method: The Geometrical Objects

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Abstract: The aim of this research is to investigate the effect of education on problem-solving skills on the Mathematical Modeling for the "Geometric Objects" unit. It was found that the topics are not supported by vital examples, they are quickly forgotten, draw less attention from the students and so students are not interested. According to the Mathematical Modeling Method, when examples are given in lessons related to daily life and students are asked to give similar examples to these, students get encouraged.It should not be overlooked that the preliminary knowledge of pre-instructional students is extremely important in terms of the planning of teaching activities. However, many teachers working in schools do not have enough knowledge about the different methods that can be used to detect unawareness or preliminary information or misconceptions, especially about preparing geometry lesson plans with the Mathematical Modeling Method.

Mathematics, especially geometry lessons, can play a key role in removing low student performance by using the Mathematical Modeling Method about problem-solving strategies, trying to solve problems, liking problem-solving, and having confidence in problem-solving, to improve students' problem-solving skills and structure.

We know that mathematics is one of the most problematic lessons in all the countries in the world. Especially when the mathematics curriculum is intensive, mathematics' teachers need to make the right choices to produce permanent solutions when choosing the teaching model. They can use the mathematical modeling method in the classroom environment to obtain good results in the academic achievement of the students and to improve their perceptions towards mathematics.

Keywords: mathematics, geometric objects, mathematical modeling, problem-solving skills

2010 Mathematics Subject Classification: 97M10, 97A30, 97D40, 97D80, 97G30.

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Other Topics (Algebra, Geometry, Topology...)

Existence and uniqueness of solutions for the first-order nonlinear integro-differential equations with three-point boundary conditions

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Abstract: Mathematical modeling of many processes occurring in the real world often leads to the study of the initial and boundary value problems for ordinary and partial differential equations. Such problems form the basis of mathematical physics. From this point of view integro-differential equations play an important and special role.

In this article, we study existence and uniqueness of solutions of nonlinear differential equations of the type

$$\dot{x}(t) = f\left(t, x(t)\right) + \int_0^t k(t,\, \tau,\, x(\tau))\, d\tau, \;\; t \in (0, T) \tag{1}$$

with three-point boundary conditions

$$Ax(0) + Bx(t_1) + Cx\left(T\right) = \alpha. \tag{2}$$

Here $A,\ B, C \in R^{n\times n}$ are constant square matrices of order n; $\alpha \in R^n$ is a given vector; $\det N \neq 0$; $N = (A + B + C), f : [0,\, T] \times R^n \rightarrow R^n$ and $k : [0,\, T\,] \times [0,\, T] \times R^n \rightarrow R^n$ are given functions, and t_1 satisfies the condition of $0 < t_1 < T$. We denote by $C\left([0,T]\,, R^n\right)$ the Banach space of all continuous functions from $[0,T]$ into R^n with the norm

$$\|x\| = \max\left\{|x\left(t\right)|\,, t \in [0,T]\right\},$$

where $|\cdot|$ is the norm in the space R^n.

We prove new existence and uniqueness results by using the fixed point theorems, and Banach contraction principle.

Similar problems for two-point boundary value problems are considered in [1-3].

Keywords: three-point boundary conditions, integro-differential equations, existence and uniqueness of solutions

2010 Mathematics Subject Classification: 34A34, 34B10

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Singularly perturbed first-order equations in complex domains that lose their uniqueness under degeneracy

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Abstract: In this paper we consider a singularly perturbed first-order equation in complex domains, the degenerate equation of which has several isolated solutions. Cases when degenerate equations have unique solutions are considered in [1-2].

Let consider the equation:

(1) $$\varepsilon z'(t,\varepsilon) = f\big(t, z(t,\varepsilon)\big),$$

where $\varepsilon > 0$ is a small parameter, C is a set of complex numbers, Ω is a simply connected domain, with the initial condition $z(t_0, \varepsilon) = z^0$ where $t_0 \in \Omega \subset C$ and is internal point. Constants independent of ε are denoted by M_0, M_1, M_2...

For $\varepsilon = 0$, from (1) we get the degenerate equation

(2) $$f\big(t, \xi, (t)\big) = 0,$$

Let the equation (2) has solutions $\xi_j(t), j = 1, 2, , n$.

Definition 1. If $\forall (k \neq m) \wedge \forall t \in \Omega (|\xi_k(t) - \xi_m(t)| > M_0)$,then $\xi_k(t)$ and $\xi_m(t)$ are said isolated solutions in Ω. U1. Let $f(t, z)$ the analytic function with respect to the variables (t, z) in some domain D the variables (t, z).

Definition 2. If:1.$\forall t \in \Omega_j \subset \Omega$ there exists $z(t, \varepsilon)$ the solution of problem (1)-(2) 2.$\forall t \in \Omega_j \big(\lim_{\varepsilon \to 0} z(t, \varepsilon) = \xi_j(t)\big)$ then the domain Ω_j are said the domain of attraction of the solution $\xi_j(t)$.

Theorem. Suppose that the condition U1 are satisfied. Then for each $\xi_j(t)$ there exists:
1. The solution $z_j(t, \varepsilon)$ of equation(1)satisfying condition $z_j(t_0, \varepsilon) = z_j^0, |\, z_j^0 - \xi_j(t_0)\,| \leq M_1 \varepsilon$.
2. The domain $\Omega_j \subset \Omega$ and $\forall t \in \Omega_j (z_j(t, \varepsilon) \to \xi_j(t) by \varepsilon)$

Keywords: singularly perturbed equations, degenerate equation, analytical functions, isolated solutions, domain of attraction

2010 Mathematics Subject Classification: 35J05,35J08, 35J25

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Characterzaton of some types of compactness and constructon ndex compactness $\leq \tau$ extensons by means of unform strutures

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Abstract: In his work [1] M. Stone noted that one of the interesting and difficult problems of General Topology is the study of all extensions of a given topological space. B. Banashevsky [2] systematized the general problems of the theory of extensions.

As is known, on every Lindelof space X is its universal uniform structure U is complete. If X_0 is all-density subspace of the space X and U_0 is a uniform structure on X_0 induced by the uniform structure of U, then the uniform space (X, U) is completion of (X_0, U_0). A uniform structure U_0 is not necessarily a universal uniform structure, but it has the special property of pre-Lindelof, and pre-Lindelof uniform structures of the space X_0 it is possible to construct all its Lindelof extensions, considering these extensions as completions of the space X_0 through to pre-Lindelof uniform structures.

In this work, through the uniform structures, the index compactness $\leq \tau$ extensions is constructed, which generalizes all Lindelof extensions, with the help of uniform structures such important compact types of topological spaces as compact, μ-compact and Lindelof spaces, also study some compact and Lindelof properties of remainders of topological spaces by means of co-coverings.

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Strongly unformly paracompactness

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Abstract:

As we know the paracompactness play an important role in the General Topology. Therefore, the finding of uniform analogues of paracompactness is an important and interesting problem in the theory of uniform spaces. Many well-known mathematicians turned to this problem and as a result different variants of uniform paracompactness of uniform spaces appeared. For example, uniform R-paracompactness in the sense of M.D. Rice [1], uniform B-paracompactness in the sense of A.A. Borubaev [2], uniform F-paracompactness in the sense of Z. Frolik [3], uniform P-paracompactness in the sense of B.A. Pasynkov [4], uniform A-paracompactness in the sense of L.V. Aparina [5], uniform I-paracompactness in the sense of D.R. Isbell [6].

In this article we show a new approach to the definition of a uniform analog of strongly paracompactness. We introduce and investigate uniform analogues of strongly paracompact and Lindelof spaces. Their connection with other properties of compactness type is studied, and the characteristics of these classes of uniform spaces are also established by means of mappings and compact Hausdorff extensions. In particular, the problem posed by A. Borubaev: what are the uniform spaces that have ω-mapping to some strongly paracompact (separable) metrizable uniform space for any (finitely additive) open covering ω?

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Existence of immovability lines of a partial mapping of Euclidean space $E_5$9

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Abstract: In domain $\Omega \subset E_5$ it is considered a set of smooth lines such that through a point $X \in \Omega$ passed one line of given set. The moving frame $\Re = (X, \overrightarrow{e}_i)$ $(i, j, k = \overline{1, 5}$ is frame of Frenet for the line ω^1 of the given set. Integral lines of the vector fields $\overrightarrow{e}_i$ are formed net Σ_5 of Frenet. There is exist the point $F_5^4 \in (X, \overrightarrow{e}_5)$ on the tangent of the line ω^5. When the point X is shifted in the domain Ω the point F_5^4 describes it's domain Ω_5^4 in E_5. It is defined the partial mapping $f_5^4 : \Omega \to \Omega_5^4$ such that $f_5^4(X) = F_5^4$.

Necessary and sufficient conditions of immovability of lines

$(X, \overrightarrow{e}_1)$, $(X, \overrightarrow{e}_2)$, $(X, \overrightarrow{e}_3)$ in partial mapping f_5^4 are proved.

Keywords: Partial mapping, cyclic net of Frenet, Frenet frame, pseudofocus, immovability of line.

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Mean value of an arithmetic function associated with the Piltz divisor function

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Abstract Let k be a fixed integer, we define the multiplicative function $D_k(n) = \frac{d_k(n)}{d_k^*(n)}$, where $d_k(n)$ is the Piltz divisor function and d_k^* is the unitary analogue function of d_k. The main purpose of this paper is using the elementary methods to study the mean value of the function $D_k(n)$.
Keywords : The Piltz divisor function, mean value, unitary analogue function.

On the Menger algebras of quasi-open multiplace maps

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Abstract: Investigation of topological spaces via groups, rings, semigroups and other algebraic structures of mappings plays an important role in modern mathematics. Many researchers have focusedtheir efforts on the abstract characterization of topological spaces by semigroups of continuous, open, quasi-open and closed mappings, particularly [2], [3]. Although these deal with the maps of one variable, the wider, but not less important class of multiplace maps are known to have various applications not only mathematics itself, but are also widly used in the theory of many-valued logics, cybernetics and general systems theory. Various natural operations are considered on the sets of multiplace functions. The main operation is the superposition, i.e., the operation which as a result of substitution of some functions into other intsead of their arguments gives a new function.

The study of functions of $n-$ variables was initiated by K. Menger [1] and continued by R.M. Dicker, B.M. Schein, W.A. Dudek and V.S. Trokhimenko. An $(n+1)-$ary operation $[\ ]$ defined on G is superassociative, if for all $x, y_1, ..., y_n, z_1, ..., z_n \in G$ the following identity holds:

$$[[x, y_1, ..., y_n]\, z_1, ..., z_n] = [x, [y_1, z_1, ..., z_n]\,, ..., [y_n, z_1, ..., z_n]]$$

An $(n+1)-$ary algebra $(G; [\ ])$satisfying the above identity is called a Menger algebra of rank n. For $n = 1$ it is an arbitrary semigroup. A function f between topological spaces X and Y is quasi-open if for any non-empty open set $U \subset X$, the interior of $f\,(U)$ in Y is non-empty. Let $Q\,(X^n, X)$ denote the Menger algebra of quasi-open functions from X^n into X with composition of functions:

$$[fg_1...g_n]\,(a_1, ..., a_n) = f\,(g_1\,(a_1, ..., a_n)\,, ..., g_n\,(a_1, ..., a_n))$$

where $a_1, ..., a_n \in X,\ f, g_1, ..., g_n \in Q\,(X^n, X)$.

The purpose of this paper is to investigate Menger algebras of quasi-open maps. It is obvious that if X and Y are homeomorphic then the Menger algebras $Q\,(X^n, X)$ and $Q\,(Y^n, Y)$ are isomorphic. If $Q\,(X^n, X)$ and $Q\,(Y^n, Y)$ are isomorphic, must X and Y be homeomorphic. In general, the answer is no. In this paper, we give an abstract characterization of Menger algebras of quasi-open functions defined on a certain class of topological spaces.

Keywords: Menger algebra, homeomorphism, quasi-open map, lattice equivalence

2010 Mathematics Subject Classification: 20N15, 54C10

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Minisymposium: Functional analysis in interdisciplinary applications

Approximation of fractional semilinear Cauchy problem in a Banach spaces

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Abstract: This talk is devoted to approximation of fractional Cauchy problem

$$\begin{aligned} &(\mathbf{D}_t^\alpha u)(t) = Au(t) + f(t, u(t)), \quad 0 < t \le T, \quad 0 < \alpha \le 1, \\ &u(0) = u^0, \end{aligned} \tag{1}$$

in a Banach space E, where $\mathbf{D}_t^\alpha$ is the Caputo-Dzhrbashyan derivative in time, the operator A generates exponentially bounded analytic and compact resolution family $S_\alpha(\cdot, A)$ and the function $f(\cdot,\cdot)$ is smooth in both arguments. The mild solution of (1) is defined as a function $u(\cdot) \in C([0,T]; E)$ which satisfies equation

$$u(t) = S_\alpha(t, A)u^0 + \int_0^t P_\alpha(t-s, A) f(s, u(s)) ds, \tag{2}$$

where for any $x \in E$ one has

$$\lambda^{\alpha-1} R(\lambda^\alpha, A)x = \int_0^\infty e^{-\lambda t} S_\alpha(t)x dt, R(\lambda^\alpha, A)x = \int_0^\infty e^{-\lambda t} P_\alpha(t)x dt, Re\lambda > \omega.$$

We consider semidiscrete approximation theorem on general approximation scheme under condition of compact convergence of resolvents.

Throughout this note we mainly used techniques from our paper [1], but the right hand side is given by the function $f(\cdot,\cdot)$ defined in different way convenient for applications. So we consider approximation of operator family $P_\alpha(\cdot)$ with weak singularity.

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Keywords: Fractional equations, Cauchy problems, semilinear problems, general approximation scheme, compact convergence of operators, index of solution

2010 Mathematics Subject Classification: 34G20, 49M25, 47D99, 65J15

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Cordoba-Cordoba type inequality on homogenous Lie groups

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Abstract: Let $s \in [0,1]$ and $x \in \mathbb{R}^n,\ n \geq 2$. In the work [1], authors show that the following inequality for the fractional Laplacian

$$2f(x)(-\Delta)^s f(x) \geq (-\Delta)^s f^2(x), \tag{1}$$

where $(-\Delta)^s$ is the fractional Laplacian, $x \in \mathbb{R}^n$ and $f(x) \in C_0^2(\mathbb{R}^n)$.

This inequality is using for the maximum principle of the quasi-geostrophic equations. Also, in the works [2] generalized the Cordoba-Cordoba inequality,

$$pf(x)(-\Delta)^s f(x) \geq (-\Delta)^s f^p(x), \tag{2}$$

where $(-\Delta)^s$ is the fractional Laplacian, $p > 0$, $x \in \mathbb{R}^n$ and $f(x) \in C_0^2(\mathbb{R}^n)$.

In the work [3], author generalized these inequalities for the fractional Laplacian. Our main aim of this talk is to establish analogues of the Cordoba-Cordoba inequality and its generalizations for the fractional sub-Laplacian on the homogenous Lie groups.

In this talk, we show an analogue of the Cordoba-Cordoba type inequality for the fractional sub-Laplacian on the homogenous Lie groups. Also, we show generalized analogueof the Cordoba-Cordoba type inequality on the homogenous Lie groups.

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Keywords: Cordoba-Cordoba inequality, fractional sub-Laplacian, homogenous Lie groups.

2010 Mathematics Subject Classification: 22E30, 43A80

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Distribution of eigenvalues of a third-order differential operator with strongly regular boundary conditions

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Abstract: This paper is devoted to the differential operator, generated by the ordinary differential equation and nonlocal boundary conditions. It is proved that the characteristic determinant of the considered spectral problem is an entire function.

The paper was supported by the state grant "The Best Teacher of the University - 2017" of the Ministry of Education and Science of the Republic of Kazakhstan

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One class of inverse problems for reconstructing the process of heat conduction from nonlocal data

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Abstract: We consider one class of inverse problems for a one-dimensional heat equation with involution and with periodic type boundary conditions with respect to a space variable.

We will consider the process which is so slow that it is described by an heat equation. Thus, this process is described by the equation

$$D_t\Phi(x,t) - \Phi_{xx}(x,t) + \varepsilon\Phi_{xx}(-x,t) = f(x) \tag{1}$$

in the domain $\Omega = \{(x,t): \ -\pi < x < \pi, \ 0 < t < T\}$. Here $f(x)$ is the influence of an external source that does not change with time; $t = 0$ is an initial time point and $t = T$ is a final one.

In this report, several variants of the operator D_t will be considered. Problems for a classical heat equation and for an equation with fractional derivatives with respect to a time variable are considered:

$$i) \quad D_t\Phi(x,t) = \Phi_t(x,t);$$

$$ii) \quad D_t\Phi(x,t) = t^{-\beta} D_{C\ t}^{\alpha}\Phi(x,t), \quad D_{C\ t}^{\alpha}\varphi(t) = I^{1-\alpha}\left[\frac{d}{dt}\varphi(t)\right], \ 0 < \alpha < 1.$$

Where $D_{C\ t}^{\alpha}$ is a Caputo derivative and $I^{1-\alpha}$ is a Riemann-Liouville fractional integral.

As the additional information we take values of the initial and final conditions of the temperature

$$\Phi(x,0) = \phi(x), \quad \Phi(x,T) = \psi(x), \quad x \in [-\pi,\pi]. \tag{2}$$

Since the wire is closed, it is natural to assume that the temperature at the ends of the wire is the same at all times:

$$\Phi(-\pi,t) = \Phi(\pi,t), \ t \in [0,T]. \tag{3}$$

Consider the process in which the temperature at one end at every time point t is proportional to the rate of change speed of the average value of the temperature throughout the wire. Then,

$$\Phi(-\pi,t) = \gamma D_t \int_{-\pi}^{\pi} \Phi(\xi,t)\,d\xi, \ t \in [0,T]. \tag{4}$$

Here γ is a proportionality coefficient.

Thus the investigated process is reduced to the following mathematical inverse problem: *Find a right-hand side $f(x)$ of the heat equation (1), and its solution $\Phi(x,t)$ subject to the initial and final conditions (2), the boundary condition (3), and condition (4).*

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Keywords: Inverse problem; heat equation; equation with involution; periodic boundary conditions; method of separation of variables

2010 Mathematics Subject Classification: 35K20, 35R30, 35R10

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On coersive solvability of a third-order differential equation with oscillating intermediate coefficient

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Abstract: In this report the sufficient conditions of well-posedness in $L_2(-\infty, +\infty)$ of the following differential equation

$$-y''' + p(x)y' + q(x)y = f(x), \tag{1}$$

with smooth, unbounded and fast oscillating coefficients are received. We give the maximal regularity estimate

$$\|y'''\|_2 + \|py'\|_2 + \|qy\|_2 \leq C\|f\|_2$$

for the solution y ($\|\cdot\|_2$ is a norm in $L_2(-\infty, +\infty)$). Here the intermediate coefficient p is not controlled by q, and $|q|$ is an unbounded function. The conditions obtained us are close to necessary.

In the case when the domain is unbounded, although the solution of the odd-order equation (1) is smooth, but it may not belong to any Sobolev space, or its belongs to the Sobolev space is not known in advance. This is one of its main differences from the Sturm-Liouville equation. So, the smoothness problems for solutions of the equation (1) are of great interest. For such equations is important to study of separability or, in other words, the maximal regularity problem.

In the case $p = 0$ the maximal regularity problem for the equation (1) and its quasilinear generalizations was investigated by M. Otelbaev, M.B. Muratbekov, K.N. Ospanov and etc. Moreover, in their works the existence of a solution was proved, and when L is linear, the uniqueness of the solution and the important spectral and approximate properties of the resolvent L^{-1} were given.

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Keywords: differential equation, unbounded domain, oscillating coeffcients, well-posedness, coercive estimate

2010 Mathematics Subject Classification: 34A30, 34B40

On a generalization of the Neumann problem for the Laplace equation

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Abstract: In the paper we investigate solvability of a fractional analogue of the Neumann problem for the Laplace equation. As a boundary operator we consider operators of fractional differentiation in the Hadamard sense. The problem is solved by reduction to the integral Fredholm equation. Theorem on existence and uniqueness of the problem solution is proved.

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On Green's function of certain boundary-value problems for the biharmonic equation

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Abstract: In this paper we study the method of constructing the Green's function of the Neumann and Robin problems analogues for biharmonic equation in the unit ball. Integral representations of the Green's function of these problems are obtained.

The work was supported by a grant from the Ministry of Science and Education of the Republic of Kazakhstan (Grant No. AP05131268).

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On the number of eigenvalues of correct restrictions and extensions for the Laplace operator

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Abstract: At the beginning of the last century J. Hadamard constructed the well-known example illustrating the incorrectness of the Cauchy problem for elliptic-type equations. If the Cauchy problem for some differential equations is correct, then it is usually a Volterra problem, that is, the inverse operator is a Volterra operator. For such differential equations there exists a set of Volterra correct problems besides the Cauchy problem. Until recently, not a single Volterra correct restriction or extension for elliptic-type equations was known.

Biyarov (see [1]) has recently proved that there are no Volterra correct restrictions and Volterra correct extensions for the Laplace operator in the two-dimensional case, it has also proved that there are no Volterra correct restrictions and extensions for a wide class of correct restrictions and extensions for the Laplace operator in the n-dimensional case.

In the present paper, we prove the theorem that the correct restrictions and extensions for the Laplace operator with discrete spectrum always have an infinite countable number of eigenvalues in the two-dimensional case.

Keywords: Laplace operator, maximal (minimal) operator, Volterra operator, correct restrictions, correct extensions, discrete spectrum, number of eigenvalues

2010 Mathematics Subject Classification: Primary 47Axx, 47A10, 47A75; Secondary 47Fxx

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An expansion result for the equation of transverse vibration of two-layered composite

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Abstract: This paper is concerned with the eigenfunctions expansions of fourth-order differential equations with impulse. Applying the Fourier series approach we find a solution of this problem and establish a uniformly convergent expansion formula in the eigenfunctions. It is necessary in applications to construct such eigenfunctions and discuss the convergence of expansions.

Keywords: Spectral problem, impulse conditions, eigenvalues, eigenfunctions

2010 Mathematics Subject Classification: 35L05, 35P10, 47A75

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A tri-harmonic Neumann function for the unit disc

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Abstract: In this paper we consider a tri-harmonic Neumann function for the unit disc. The Neumann problem is well studied for harmonic functions and solved under certain conditions via the Neumann functions, sometimes also called Green function of second kind [1]. A tri-harmonic Neumann function is constructed in an explicit way for the unit disc of the complex plane by convoluting the harmonic with a bi-harmonic Neumann function. With this Neumann function an integral representation formula is developed for the try-harmonic operator.

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About the Generalized Dirichlet – Neumann Problem for an Elliptic Equation of High Order

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Abstract: For the elliptic equation $2l-$ th order with constant (and only) real coefficients considered boundary value problem of the job normal derivatives the $(k_j-1)-$ order, $j = 1, \ldots, l$, where $1 \leq k_1 < \ldots < k_l \leq 2l-1$. When $k_j = j$ it moves to the Dirichlet problem, and when $k_j = j+1$ - in the Neumann problem. In this paper, the study is carried out in space $C^{2l,\mu}(\overline{D})$. The sufficient condition of the Fredholm tasks and present a Formula for its index.

Let $s \in [0,1]$ and $x \in \mathbb{R}^n$, $n \geq 2$. In the work [1], authors show that the following inequality for the fractional Laplacian

$$2f(x)(-\Delta)^s f(x) \geq (-\Delta)^s f^2(x), \tag{1}$$

where $(-\Delta)^s$ is the fractional Laplacian, $x \in \mathbb{R}^n$ and $f(x) \in C_0^2(\mathbb{R}^n)$.

This inequality is using for the maximum principle of the quasi-geostrophic equations. Also, in the works [2] generalized the Cordoba-Cordoba inequality,

$$pf(x)(-\Delta)^s f(x) \geq (-\Delta)^s f^p(x), \tag{2}$$

where $(-\Delta)^s$ is the fractional Laplacian, $p > 0$, $x \in \mathbb{R}^n$ and $f(x) \in C_0^2(\mathbb{R}^n)$.

In the work [3], author generalized these inequalities for the fractional Laplacian. Our main aim of this talk is to establish analogues of the Cordoba-Cordoba inequality and its generalizations for the fractional sub-Laplacian on the homogenous Lie groups.

In this talk, we show an analogue of the Cordoba-Cordoba type inequality for the fractional sub-Laplacian on the homogenous Lie groups. Also, we show generalized analogueof the Cordoba-Cordoba type inequality on the homogenous Lie groups.

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Keywords: Cordoba-Cordoba inequality, fractional sub-Laplacian, homogenous Lie groups.

2010 Mathematics Subject Classification: 22E30, 43A80

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On a certain class of pseudo-differential operators and related equations

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Abstract: To extend obtained results on Fredholm properties of pseudo-differential operatos on manifolds with non-smooth boundaries [1,2] we start from studying model pseudo-differential operators

$$(Au)(x) = \int\limits_{\mathbb{R}^m}\int\limits_{\mathbb{R}^m} A(x,\xi)e^{i(x-y)\xi}u(y)dyd\xi, \quad x \in \mathbb{R}^m,$$

in local Sobolev–Slobodetskii space $H^{s(x)}(\mathbb{R}^m)$ with finite value

$$||u||_{s(x)} \equiv \left(\int\limits_{\mathbb{R}^m} (1+|\xi|)^{2s(x)}|\tilde{u}(\xi)|^2 d\xi\right)^{1/2},$$

where $\tilde{u}$ denotes the Fourier transform of the function u.

Some Fredholm properties for elliptic pseudo-differential operator are obtained for a whole space $\mathbb{R}^m$ and a half-space $\mathbb{R}^m_+$.

Keywords: pseudo-differential operator, variable order, invertibility, Fredholm property

2010 Mathematics Subject Classification: 47G30, 35S15, 58J05

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Spectral Properties of the Sturm-Liouville Operator with a Parameter That Changes Sign and Their Usage to the Study of the Spectrum of Differential Operators of Mathematical Physics Belonging to Different Types

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Abstract: This report is devoted to the spectral properties of the Sturm-Liouville operator with a parameter that changes sign and their application to the study of the spectral properties of differential operators of elliptic, hyperbolic and parabolic types.

Consider the differential operator

$$Lu = k\left(y\right)\frac{\partial^2 u}{\partial x^2} - \frac{\partial^2 u}{\partial y^2} + b\left(y\right)u_x + m\left(y\right)u \tag{1}$$

originally defined on $C_0^\infty\left(R^2\right)$, where $k\left(y\right)$ is a piecewise continuous and bounded function changing sign in R, $C_0^\infty\left(R^2\right)$ is the set of infinitely differentiable and compactly supported functions, the coefficients $b\left(y\right), m\left(y\right)$ are continuous functions in R and bounded in each compact set.

It is easily verified that the operator can belong to different types, depending on the sign of the coefficient $k(y)$.

For instance, let $k\left(y\right) = -1$, then the operator L is of elliptic type. If however, $k\left(y\right) = 1$ the operator will be hyperbolic. In the case $k\left(y\right) = 0$, the operator L is of parabolic type.

It is known that the three types of partial differential operators considered here play a special role in mathematical physics. And finally, the type of the operator L may be different at different points if the function $k\left(y\right)$ changes sign in R. We note that such operator is called an operator of mixed type.

Further, let $u\left(x,y\right) \in C_0^\infty\left(R^2\right)$, then it is easy to see that after applying the Fourier transformation with respect to x the operator L takes the form:

$$L_t\hat{u} = -\hat{u}_{yy}^{''} + \left(-k\left(y\right)t^2 + itb\left(y\right) + m\left(y\right)\right)\hat{u}, \tag{2}$$

where $\hat{u}\left(t,y\right) = F_{x\to t}u\left(x,y\right) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} u\left(x,y\right)e^{-itx}dx$.

Hence it is easy to notice that questions on the existence of the resolvent and spectral properties of the operator L transfer into problems on the existence and spectral properties of the Sturm-Liouville operator with a potential involving a parameter with changing sign.

The following results: 1) the existence of the resolvent, 2) the criterion of the spectrum discreteness, 3) two-sided estimates of approximation numbers (s-numbers) are obtained for operator 2. These results are applied to the study of the spectral properties of differential operators of elliptic, parabolic and hyperbolic types.

Throughout this note we mainly use techniques from our works [1-5].

Keywords: resolvent, spectrum, operator of mixed type, spectral properties, Sturm-Liouville operator with a parameter

2010 Mathematics Subject Classification: 34B24; 34L05; 35M10; 47A10

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About Volterra's nonlocal boundary value problem with displacement for the wave equation

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Abstract: Recently the interest in boundary value problems of the wave equation sharply have increased due to the growing needs of Geophysics and Seismology. In this paper we show that the number of boundary value problems for the wave equation, taken inside the characteristic quadrangle are Volterra. The main results of this work are presented by following theorems:

Theorem 5.1. *If $\alpha^2 - \beta^2 \neq 0$, then a nonlocal boundary value problem with displacement*

$$Lu = u_{xy}(x,y) = f(x,y), (x,y) \in \Omega; \tag{1}$$

$$\alpha \cdot u(0,y) + \beta \cdot u(1, 1-y) = 0, \tag{2}$$

$$\alpha \cdot u(x,0) + \beta \cdot u(1-x, 1) = 0, \tag{3}$$

is strongly solvable, and the inverse operator L^{-1} to the strong operator of problem (1)–(3) is completely continuous and Volterra.

Theorem 5.2. *If $\alpha^2 - \beta^2 \neq 0$, then nonlocal boundary value problem*

$$Lu(x,y) = u_{xy}(x,y) = f(x,y), \qquad f(x,y) \in L^2(\Omega), \tag{4}$$

$$\alpha u(0,y) + \beta u(y,0) = 0, \tag{5}$$

$$\alpha u(x,1) + \beta u(1,x) = 0 \tag{6}$$

is strongly solvable in space $L^2(\Omega)$, and the operator L^{-1} inverse to the strong operator of problem (4)–(6) is completely continuous and Volterra.

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Keywords: Volterra operators, Goursat problem, wave equation, similarity operators

2010 Mathematics Subject Classification: 35L05, 34L10, 35L20

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On the Spectral Properties of a Wave Operator Perturbed by a Lower-Order Term

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Abstract: The incorrectness of the minimal wave operator is well known, since zero is an infinite-to-one eigenvalue for it. As our study showed, the situation changes if the operator is perturbed by a low-order term containing the spectral parameter as a coefficient, and eventually the problem takes the form of a bundle of operators. The resulting bundle of operators is easily factorized by first-order functional-differential operators whose spectral properties are easily studied by the classical method of separation of variables. Direct application of the method of separation of variables to the original bundle of operators encounters insurmountable difficulties. In this work we consider the following spectral problem:

$$u_{xx} - u_{yy} = -2\lambda u_x + \lambda^2 u, \tag{1}$$

$$u|_{y=0} = 0, \quad u|_{x=0} = \alpha\, u|_{x=1}, \quad |\alpha| = 1. \tag{2}$$

We investigated the spectral properties of task(1)-(2) and obtain the eigenvalues and corresponding eigenfunctions, which form an orthonormal basis of the space $L^2(\Omega)$. The spectrum of the beam of operators (1)-(2) consists of two series of infinite eigenvalues.

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Keywords: deviating argument, strong solvability, spectrum, beam of operators

2010 Mathematics Subject Classification: 35L20, 35D35, 47A10

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Compactness Of The Commutators Of Riesz Potential On Global Morrey-type Space

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Abstract: In this paper we obtain the sufficient conditions of compactness of the commutator for the Riesz potential $[b, I_\alpha]$ in global Morrey-type spaces $GM^w_{p\theta}$.

Let $1 \le p \le \infty$, w be a measurable non-negative function on $(0,\infty)$. The Global Morrey-type space $GM^w_{p\theta} \equiv GM^w_{p\theta}(\mathbb{R}^n)$ is defined as the set of all functions $f \in L^{loc}_p(\mathbb{R}^n)$ with finite quasi-norm $\|f\|_{GM^w_{p\theta}} \equiv \sup\limits_{x\in\mathbb{R}^n} \left\| w(r)\, \|f\|_{L_p(B(x,r))} \right\|_{L_\theta(0,\infty)}$, where $B(t,r)$ the ball with center at the point t and of radius r.

For the function $b \in L_{loc}(\mathbb{R}^n)$ we denote by M_b the multiplication operator $M_b f = bf$, where f is a measurable function. Then the commutator for the Riesz potential I_α and the operator M_b is defined by the equality $[b, I_\alpha] = M_b I_\alpha - I_\alpha M_b = C_{n,\alpha} \int\limits_{\mathbb{R}^n} \frac{[b(x)-b(y)]f(y)}{|x-y|^{n-\alpha}} dy$. It is said that the function $b(x) \in L_\infty(\mathbb{R}^n)$ belongs to the space $BMO(\mathbb{R}^n)$ if $\|b\|_* = \sup_{Q\subset\mathbb{R}^n} \frac{1}{|Q|} \int\limits_Q \left|b(x) - b_Q\right| dx = \sup_{Q\in\mathbb{R}^n} M(b,Q) < \infty$, where Q - cub $\mathbb{R}^n$ and $b_Q = \frac{1}{|Q|} \int\limits_{\mathbb{R}^n} f(y)dy$.

We denote by $VMO(\mathbb{R}^n)$ the BMO -closure of the space $C^\infty_0(\mathbb{R}^n)$, where $C^\infty_0(\mathbb{R}^n)$ the set of all functions in $C^\infty(\mathbb{R}^n)$ with compact support.

Theorem 2. Let $0 < \alpha < n(1-\frac{1}{q})$, $1 \le p_1 < p_2 < \infty$, $\alpha = n(\frac{1}{p_1} - \frac{1}{p_2})$, or $1 \le\ p_1 < \infty$, $1 \le p_2 < \infty$ and $n(\frac{1}{p_1} - \frac{1}{p_2}) < \alpha < \frac{n}{p_1}$, $\alpha < n(1-\frac{1}{p_2})$, $0 < \theta_1 \le \theta_2 \le \infty$, $\theta_1 \le 1$, $w_1 \in \Omega_{p_1\theta_1}$, $w_2 \in \Omega_{p_2\theta_2}$, and let $\left\| w_2(r) \frac{r^{\frac{1}{p_2}}}{(t+r)^{\frac{n}{p_1}-\alpha}} \right\|_{L_{\theta_2(0,\infty)}} \le c\,\|w_1(r)\|_{L_{\theta_1(t,\infty)}}$ for all $t > 0$, where $c > 0$ not depend on t, and $b \in VMO(\mathbb{R}^n)$. In addition, let the commutator $[b, I_\alpha]$ bounded from $GM^{w_1}_{p_1\theta_1}$ to $GM^{w_2}_{p_2\theta_2}$. More over, let $\|[b, I_\alpha]\|_{GM^{w_2}_{p_2\theta_2}} \le \|b\|_* \|f\|_{GM^{w_1}_{p_1\theta_1}}$ Then the commutator $[b, I_\alpha]$ is a compact operator from $GM^{w_1}_{p_1\theta_1}$ to $GM^{w_2}_{p_2\theta_2}$

Keywords: Compactness, Commutators, Riesz Potential, Global Morrey-type Space

2010 Mathematics Subject Classification: 35Q79, 35K05, 35K20

Sharp by order two-sided estimates for one class of function series

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Abstract: In this work we consider the function series

$$J := J_t = \sum_{i=t}^{\infty} \frac{1}{\prod_{j=t}^{i}(1+r_j)}, \ t = 0, 1, 2, \dots, \tag{1}$$

with function $r_t \geq 0$, $\prod_{j=t}^{t+T-1}(1+r_j) < \infty$, $T = 1, 2, \dots$. We find sharp by order two-sided estimates for the function J.

We note that the functional series (1) was obtained by Kerimkhulle S. and Aitkozha Zh. in [1] as a solution of the first order linear difference equation. Also note that in [2], Lukachev M. and Shuster L. obtained find sharp by order two-sided estimates in case the integral functional (1). For a proof of main results is obtained by Otelbayev (see [3]) variational methods for localization.

Keywords: function series, upper estimate, lower estimate, sharp by order two-sided estimate

2010 Mathematics Subject Classification: 39A10, 40A05, 40A10

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Degenerate operators for ordinary differential equations

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Abstract: We consider the spectral problem for operators generated by ordinary differential equations. A degenerate operator means an operator whose point spectrum fills the whole complex plane, or a resolvent set fills the whole complex plane.

Conditions are found for the coefficients of the general differential equation for which the corresponding operator is degenerate.

Theorem: Let the operator L generated in $L_2(0,1)$ by the differential expression $l(y) = y^{(n)} + \sum_{j=1}^{n} p_j(x) y^{(n-j)}$, and the boundary conditions $U_j(y) = y^{(j-1)}(0) - \alpha \cdot \alpha_2^{(j-1)} y^{(j-1)}(1) = 0$, $j = 1, 2, \ldots, n$, where $\alpha_k = e^{i\frac{2\pi(k-1)}{n}}$, $k = 1, 2, \ldots, n$, being the n-th roots of unity with $\alpha_1 = 1$. If the condition

$$p_k(x) = \alpha_2^k p_k(1 + \alpha_2 x), \quad k = 1, 2, \ldots, n, \ \ 0 \leq x \leq 1,$$

is satisfied, then the operator L is degenerate, that is, the characteristic determinant $\Delta_L(\rho) \equiv const$. Furthermore, $\Delta_L(\rho) = 1 - \alpha^n$, and if $\alpha^n \neq 1$, then L^{-1} is a Volterra operator; if $\alpha^n = 1$, then $\rho(L) = \varnothing$ and $\sigma(L) = \mathbb{C}$.

Keywords: ordinary differential equations, degenerate operator, spectrum, resolvent set

2010 Mathematics Subject Classification: Primary 34L05

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Generalized three-dimensional singular integral equation by tube domain

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Abstract: In this work, we investigate one class of three- dimensional complex integral equation by tube domains, ave in lower basis and lateral surface and may have singularity.

Let Ω denote the tube domain $\Omega = \{(z,t) : a < t < b, |z| < R\}$. Lower ground this cylinder denote by $D = \{t = a, |z| < R\}$ and lateral surface denote by $S = \{a < t < b, |z| = R\}, z = x + iy$. In Ω we shall consider the following integral equation

$$\varphi(t,z) + \int_a^t \frac{K_1(t,\tau)}{\tau - a}\varphi(\tau,z)d\tau + \frac{1}{\pi}\iint\limits_D \frac{\exp[i\theta]K_2(r,\rho)}{(R-\rho)(s-z)}\varphi(t,s)ds$$

$$+\frac{1}{\pi}\int_a^t \frac{d\tau}{\tau - a}\iint\limits_D \frac{K_3(t,\tau;r,\rho)}{(R-\rho)(s-z)}\exp[i\theta]\varphi(\tau,s)ds = f(t,z), \qquad (1)$$

where $\theta = \arg s, s = \xi + i\eta, ds = d\xi d\eta, \rho^2 = \xi^2 + \eta^2, r^2 = x^2 + y^2,\ K_1(t,\tau) = \sum\limits_{j=1}^{n} A_j \ln^{j-1}(\frac{t-a}{\tau-a}), K_2(r,\rho) = \sum\limits_{l=1}^{m} B_l \ln^{l-1}(\frac{R-r}{R-\rho}), K_3(t,\tau;r,\rho) = K_1(t,\tau)K_2(r,\rho), A_j(1 \le j \le n), B_l(1 \le l \le m)$ are given constants, $f(t,z)$ are given function, $\varphi(t,z)$ unknown function. In depend from the roots of the characteristics equations

$$\lambda^n + \sum_{j=1}^{n} A_j(j-1)!\lambda^{n-j} = 0, \mu^m + \sum_{l=1}^{m} B_l(l-1)!\mu^{m-j} = 0$$

obtained representation the manifold solution of the integral equation (1), by m arbitrary functions $\Phi_l(t,z)(1 \le l \le m)$ analytically by variables z and continuously by variables t and n arbitrary function $C_j(z)$ $(1 \le j \le n)$ continuously by variables z.

Keywords: tube domain; singular kernels; manifold solution; logarithmic singularity.

2010 Mathematics Subject Classification: 44A15, 35C10, 45E10

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On a boundary condition of Bitsadze-Samarskii for the Lavrent'ev-Bitsadze equation

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Abstract: This report is devoted to study the Bitsadze-Samarskii boundary condition for the Lavrent'ev-Bitsadze equation. In the domain D we consider the two-dimensional integral potential:

$$u = L_B^{-1} f = \begin{cases} \Delta_N^{-1} f^-(x_1, x_2), & (x_1, x_2) \in D^-, \\ \Box^{-1} f^+(x_1, x_2) + \Box_0^{-1} \tau(x_1, x_2) + \Box_1^{-1} \nu(x_1, x_2), & (x_1, x_2) \in D^+, \end{cases}$$

where

$$\Delta_N^{-1} f^-(x_1, x_2) = \int_{D^-} \varepsilon_2^-(x_1 - \xi_1, x_2 - \xi_2) f^-(\xi_1, \xi_2) d\xi_1 d\xi_2 \tag{1}$$

the two-dimensional Newton potential and the wave potential:

$$\Box^{-1} f^+(x_1, x_2) = \int_0^b d\xi_2 \int_0^1 \varepsilon_2^+(x_1 - \xi_1, x_2 - \xi_2) f^+(\xi_1, \xi_2) d\xi_1 \tag{2}$$

and surface wave potentials:

$$u_\tau(x_1, x_2) = \Box_0^{-1} \tau(x_1, x_2) = \varepsilon_2^+ * \tau(x_1) \delta^{(/)}(x_2), \tag{3}$$

$$u_\nu(x_1, x_2) = \Box_1^{-1} \nu(x_1, x_2) = \varepsilon_2^+ * \nu(x_1) \delta(x_2). \tag{4}$$

When $f(x_1, x_2) \in C^{1+\alpha}(\overline{D})$ the integral operator $L_B^{-1} f(x_1, x_2) \in C^{1+\alpha}(\overline{D}) \cap C^{2+\alpha}(\overline{D^-}) \cap C^{2+\alpha}(\overline{D^+})$ satisfies the equation of mixed (elliptic-hyperbolic) type:

$$L_B u(x_1, x_2) = \mathrm{sgn} x_2 u_{x_1 x_1}(x_1, x_2) - u_{x_2 x_2}(x_1, x_2) = f(x_1, x_2), \quad (x_1, x_2) \in D, \tag{5}$$

which is called the Lavrent'ev-Bitsadze equation. In this paper, by using techniques from our works [1,2] we found the boundary conditions for the Bitsadze integral operator $L_B^{-1} f$.

Keywords: Newton potential, wave potential, surface wave potentials, Bitsadze-Samarskii boundary condition, Lavrent'ev-Bitsadze equation

2010 Mathematics Subject Classification: 35J05, 35J25, 35L20

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Numerical Solution of a Two Dimensional Elliptic-Parabolic Equation with Dirichlet-Neumann Condition

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Abstract: In the present paper, a two dimensional elliptic-parabolic equation with Dirichlet-Neumann boundary condition is studied. The first and second order of accuracy difference schemes for the numerical solution of this problem are presented. Illustrative numerical results of these difference schemes are provided by using a procedure of modified Gauss elimination method.

Keywords: Elliptic-Parabolic equation, nonlocal boundary value problems, stability, difference scheme

2010 Mathematics Subject Classification: 35J15, 35K10, 35M12, 65M12

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On the unique solvability of a nonlocal problem with integral condition for the impulsive partial differential equation third order

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Abstract: The nonlocal problem with integral condition for the impulsive partial differential equation third order is investigated. Conditions of the unique solvability to nonlocal problem are established in the terms of initial data.

In this report, on $\Omega = [0,T] \times [0,\omega]$ the nonlocal problem with integral condition for the impulsive partial differential equation is considered

$$u_{txx} = A(t,x)u_{xx} + B(t,x)u_{tx} + C(t,x)u_x + f(t,x), \qquad t \neq t_i, \tag{1}$$

$$P_i(x)u(t_i+0,x) - S_i(x)u(t_i-0,x) = \varphi_i(x), \qquad i = \overline{1,m}, \quad x \in [0,\omega], \tag{2}$$

$$L(x)u_{xx}(0,x) + \sum_{j=1}^{k} \int_{t_{j-1}}^{t_j} M_j(t,x)u_{xx}(t,x)dt = \varphi_0(x), \quad x \in [0,\omega], \tag{3}$$

$$u(t,0) = \psi_1(t), \qquad u_x(t,x)\big|_{x=0} = \psi_2(t), \qquad t \in [0,T], \tag{4}$$

where $u(t,x)$ is unknown function, the functions $A(t,x)$, $B(t,x)$, $C(t,x)$, and $f(t,x)$ are continuous on Ω, the functions $P_i(x)$, $S_i(x)$, $\varphi_i(x)$ are twice continuously differentiable on $[0,\omega]$, $i = \overline{1,m}$, $0 < t_1 < t_2 < \ldots < t_m \leq T$, the functions $L(x)$, $\varphi_0(x)$, $M_j(t,x)$ are continuous on $[0,\omega]$, Ω, respectively, $j = \overline{1,k}$, the functions $\psi_1(t)$ and $\psi_2(t)$ are continuously differentiable on $[0,T]$.

Problem (1)–(4) is reduced to equivalent problem, consisting of nonlocal problem with integral condition for impulsive hyperbolic equation second order and functional relations. Algorithm of finding approximate solution to researching problem is constructed and the its convergence is proved. The conditions of classical solvability to problem (1)–(4) are established by method of introduction additional functional parameters [1, 2].

Keywords: partial differential equation, nonlocal problem, impulse effect

2010 Mathematics Subject Classification: 35G16, 35L20, 35R12

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On the spectral properties of a differential operator arising in the theory of Brownian motion

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Abstract: It is well known that in stochastic processes is widely used the following generalized Ornstein-Uhlenbeck operator

$$Au = -\Delta u + \nabla u \cdot b(x) + c(x)u,$$

where $x \in R^n$. So, A is the generator of the transition semigroup of the stochastic process, which determines the n-dimensional Brownian motion with identity covariant matrix.

The vector-valued function b is called a drift. If $b = 0$, then A is the Schrodinger operator, which has been systematically studied for a long time in connection with applications in quantum mechanics. If $b \neq 0$ and it is unbounded, then the properties of A are significally differ. For example, the well-posedness and spectral properties of A depend on the relationship between the growth at infinity of $|b|$ and the norm of c. For more details, we refer to papers of A. Lunardi and V. Vespri (1997), G. Metafune (2001) and P.J. Rabier (2005).

In this work we discuss the discreteness spectrum criterion of an folloving one-dimentional operator

$$Ly = -y'' + r(x)y' + q(x)y,$$

given in $L_p(R)$ $(1 < p < \infty)$, where r is a continuously differentiable function, and q is a continuous function, and the growth of r not depends on q. To get the desired result, we first give the conditions for invertibility of L and describe its domain $D(L)$.

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Keywords: differential operator, unbounded coefficients, well-posedness, discreteness of spectrum

2010 Mathematics Subject Classification: 34L05

Duhamel principle for the time-fractional diffusion equation in unbounded domain

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Abstract: In this paper we establish a fractional Duhamel principle for the time-fractional diffusion equation

(1) $$u_t(x,t) - \frac{\partial^2}{\partial x^2} D_t^{1-\alpha} u(x,t) = f(x,t),\ 0 < \alpha < 1,\ x \in \mathbb{R},\ t > 0,$$

with the initial condition

(2) $$u(x,0) = u_0(x),\ x \in \mathbb{R},$$

where $u_0(x) \in L^p(\mathbb{R})$, $p \geq 1$, $f(x,t)$ is a continuously differentiable function and $f(x,0) = 0$ and $D^{1-\alpha}$ represents the following Riemann-Liouville fractional derivative of order $1-\alpha$

$$D_t^{\alpha} u(x,t) = \frac{1}{\Gamma(1-\alpha)} \frac{\partial}{\partial t} \int_0^t (t-s)^{-\alpha} f(s) ds.$$

In [1,2] generalized the classical Duhamel principle for the Cauchy problem to general inhomogeneous fractional distributed differential-operator equations. **Keywords:** Duhamel Principle, diffusion equation, fractional derivative, Green's function.

2010 Mathematics Subject Classification: 31A30, 31B30, 35J40.

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Criterion for the unconditional basicity of the root functions related to the second-order differential operator with involution

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Let L be any operator related to the operation of the form

$$Lu \equiv -u''(x) + \alpha u''(-x) + q(x)u(x) + q_v(x)u(v(x)),\ -1 < x < 1,\ (1)$$

and defined on a dense in $L_2(-1,1)$ domain $D(L)$. The operation (1) contains the argument's transform $v_0(x) = -x$, in its main term. This transform is called a simple involution (reflection) of the segment $[-1;1]$ and also n arbitrary involution $v(x)$ in the lower term. The parameter α in (1) belongs to (-1;1), the coefficients $q(x)$ and $q_v(x)$ are arbitrary and complex-valued integrable on $[-1;1]$ functions, the involution $\nu(x)$ is any absolutely continuous function which has an essentially bounded derivative on $[-1;1]$.

The particular form of the domain $D(L)$ will not be refined below; the operator L can be generated by the functional-differential operation (3.1), for example, with some boundary conditions on the segment $[-1;1]$. We only assume that the domain $D(L)$ contains only functions that, together with their first derivatives, are absolutely continuous on the interval $(-1;1)$, while the root functions of the operator L are considered as regular solutions of the corresponding equations with a spectral parameter.

Following Il'in [1], an eigenfunction (or a root function of the zero order) $u(x)$, that corresponds to the operator (3.1) and an eigenvalue $\lambda \in \mathbb{C}$ is defined as an arbitrary non trivial solution of the equation $Lu = \lambda u$. Here and throughout, a regular solution of the equation $Lu = f$ with a given right-hand side $f \in L_1(-1,1)$ is understood to be an arbitrary function $u(x)$ from the class $W_1^2(-1,1) \cap L_2(-1,1)$, that satisfies this equation almost everywhere on $(-1;1)$.

Let $\tilde{u}(x)-$ be a root function of order (k $-$ 1) $(k \geq 1)$, corresponding to an eigenvalue λ. Then the regular solution of the equation $Lu = \lambda u - \tilde{u}$ will be called its counterpart root (associated) function of order k.

For each eigenvalue $\lambda \in C$, we have there by defined a chain of root functions $u_k(x;\lambda)$, $k \geq 0$ that satisfy the

$$Lu_k(x;\lambda) = \lambda u_k(x;\lambda) - sgnk \cdot u_{k-1}(x;\lambda), \qquad (2)$$

moreover, $u_0(x;\lambda) \not\equiv 0$ on $(-1;1)$.

Any count able set $\Lambda = \{\lambda\} \subset \mathbb{C}$ defines the system of root functions

$$U = \{u_k(x;\lambda) \,|k = 0,...,m(\lambda), \lambda \in \Lambda\};$$

here then on negative integer $m(\lambda)$ will be called the rank of the corresponding eigenfunction $u_0(x;\lambda)$.

Left the system U satisfy the following conditions A:

A1) the system U is complete and minimal in $L_2(-1,1)$;

A2) a system V that is biorthogonally adjoint to U consists of root functions $v_l(x;\lambda^*)$, $l = 0,...,m(\lambda^*)$, $\lambda^* \in \overline{\Lambda}$, $m(\lambda^*) = m(\lambda)$, (in the above-defined sense) of the formal adjoint operation

$$L^*v = -v''(x) + \alpha v''(-x) + \overline{q(x)}v(x) - v'(x)\overline{q_v(v(x))}v(v(x)), \qquad (3)$$

and the relation $\left(u_k(\cdot;\lambda), v_{m(\lambda)-l}(\cdot;\lambda^*)\right) = 1$ is valid if and only if $k = l$ and $\lambda^* = \overline{\lambda}$; while in the remaining cases the inner product on the left-hand side in relation (3.4) is zero;

A3) the ranks of the eigenfunctions are uniformly bounded: $\sup\limits_{\lambda \in \Lambda} m(\lambda) < \infty$

and the condition that the set Λ belongs to the Carleman parabole is satisfied $\sup\limits_{\lambda \in \Lambda} \left|\mathrm{Im}\sqrt{\lambda}\right| < \infty$;

A4) the following uniform estimate of the "sum of units" is valid:

$$\sup_{\beta \geq 1} \sum_{\lambda \in \Lambda: \left|\mathrm{Re}\sqrt{\lambda - \beta}\right| \leq 1} 1 < \infty.$$

Theorem 1. Let the conditions 1-4 be satisfied and let the involution $\nu(x)$ occurring in (1) be an arbitrary continuous function with the derivative that is essentially bounded on the segment $[-1,1]$. Then each of the systems U and V of root functions of the operators (1) and (3), respectively, forms an unconditional basis in $L_2(-1,1)$ if and only if the uniform estimate of the product of norms $\|u_k(\cdot;\lambda)\|_2 \cdot \left\|\upsilon_{m(\lambda)-k}\left(\cdot;\bar{\lambda}\right)\right\|_2 \leq M$holds for all $\mathrm{k} = 0,\ldots,\mathrm{m}(\lambda)$and $\lambda \in \Lambda$The main theorem is complemented with the proof of the necessity of condition A4 in the case where the involution $\nu(x)$in the operator (1) is a reflection.

Theorem 2. Let the condition A3 be satisfied and, in addition, let $\nu(\mathrm{x}) = -\mathrm{x}$ If the system of root functions U that is normed in $\mathrm{L}_2(-1,1)$ possesses the Bessel property, then the uniform estimate of the "sum of units" A4 is valid.

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Compactness results for the resolvent of a singular second order hyperbolic operator with a mixed derivative

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Abstract: The resolvent compactness question for differential operators defined in an unbounded domain was studied, mainly, in the one-dimensional case, or in the case of an elliptic operator. There are works [1] and [2] devoted to the discreteness of the spectrum and other important properties of the resolvent of hyperbolic and mixed type operators. However, in these papers were considered Â operators given in the Hilbert and Lebesgue spaces. At the same time, a number of practical problems lead to singular differential operators in the class of continuous functions.

Let $\overline{\Omega} = [0;\omega] \times (-\infty;\infty)$. We denote by $C_*(\overline{\Omega}, R^n)$ the space with norm

$$\|U\| = \sup_{(x,t)\in\overline{\Omega}} |U(x,t)|$$

of bounded functions $U : \overline{\Omega} \to R^n$ that are continuous on $t \in R$ for any fixed $x \in [0, \omega]$ and uniformly with respect to $t \in R$ continuous on $x \in [0, \omega]$.

In this work in $C_*(\overline{\Omega}, R^n)$ we consider the following hiperbolic operator:

$$LU = \frac{\partial^2 U}{\partial t \partial x} - A(x,t)\frac{\partial U}{\partial x} - B(x,t)\frac{\partial U}{\partial t} - C(x,t)U.$$

We present conditions for the bounded invertibility of L and the compactness of the inverse operator L^{-l}. Here $A(x,t)$, $B(x,t)$, $C(x,t)$ are the continuous $(n\times n)-$ matrixes.

Keywords: singular hiperbolic operator, space of bounded functions, well-posedness, resolvent, compactness.

2010 Mathematics Subject Classification: 35L81, 47F05, 47A10

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Structure of essential spectra and discrete spectrum of the energy operator of five-electron systems in the Hubbard model. Fifth doublet state

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Abstract: We consider the energy operator of five-electron systems in the Hubbard model and describe the structure of essential spectrum and discrete spectrum of the system in the fifth doublet state. The Hamiltonian of the chosen model has the form

$$H = A\sum_{m,\gamma} a^+_{m,\gamma}a_{m,\gamma} + B\sum_{m,\tau,\gamma} a^+_{m,\gamma}a_{m+\tau,\gamma} + U\sum_{m} a^+_{m,\uparrow}a_{m,\uparrow}a^+_{m,\downarrow}a_{m,\downarrow}.$$

Here, A is the electron energy at a lattice site, B is the transfer integral between neighboring sites (we assume that $B>0$ for convenience), $\tau=\pm e_j, j=1,2,...,\nu$, where e_j are unit mutually orthogonal vectors, which means that summation is taken over the nearest neighbors, U is the parameter of the on-site Coulomb interaction of two electrons, γ is the spin index,$\gamma=\uparrow$ or $\gamma=\downarrow$, and $a^+_{m,\gamma}$ and $a_{m,\gamma}$ are the respective electron creation and annihilation operators at a site $m\in Z^\nu$. Let ${}^5\widetilde{H}^d_{1/2}$ denote the operator of fifth doublet state of five-electron systems and W and m are the positive numbers.

Theorem. Let $\nu=3$. **a).** *If* $U<-\frac{4B}{m}$, $m<\frac{4}{3}W$, *or* $U<-\frac{3B}{W}$, $m>\frac{4}{3}W$, *then the essential spectrum of operator* ${}^5\widetilde{H}^d_{1/2}$ *is exactly the union of seven segments and the discrete spectrum of* ${}^5\widetilde{H}^d_{1/2}$ *consists of no more one point.*

b). *If* $-\frac{3B}{W}\leq U<-\frac{4B}{m}$, *then the essential spectrum of* ${}^5\widetilde{H}^d_{1/2}$ *is the union of four segments and the discrete spectrum of* ${}^5\widetilde{H}^d_{1/2}$ *is empty.*

c). *If* $-\frac{4B}{m}\leq U<0$, $m>\frac{4}{3}W$, *or* $-\frac{3B}{W}\leq U<-\frac{3B}{m}$, *and* $m<\frac{4}{3}W$, *then the essential spectrum of* ${}^5\widetilde{H}^d_{1/2}$ *is the union of two segments and the discrete spectrum of* ${}^5\widetilde{H}^d_{1/2}$ *is empty.*

d). *If* $-\frac{4B}{m}\leq U<0$, $m>\frac{4}{3}W$, *or* $-\frac{3B}{W}\leq U<0$, $m<\frac{4}{3}W$, *then the essential spectrum of operator* ${}^5\widetilde{H}^d_{1/2}$ *is the single segment and the discrete spectrum of operator* ${}^5\widetilde{H}^d_{1/2}$ *is empty.*

Keywords: Hubbard Model, Essential and Discrete spectrum, five-electron systems, sextet state, quartet state, doublet states,bound and antibound states.

2010 Mathematics Subject Classification: 47Axx, 46XX, 46L60,11F72, 35Pxx,62M15,58C40

Reduction of Navier-Stokes Equation to a Linear Equation

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Abstract: In this article, we provide two theorems on pointwise coincidence between solutions of Navier-Stokes equation and solutions of standard linear second order parabolic equations with the same data. We show that the convection, the pressure, and the external forces (if applied) are governed by some sort of balance independent of the equation that governs the solution itself. In light of the well establishment of the theory of existence, regularity and uniqueness of linear second order parabolic equations, this result represents an important step to fully understand the qualitative properties of the solutions to Navier-Stokes equation.

Details: The initial profile is $\mathbf{v}_0$ and the boundary conditions (in case of bounded domains) are denoted by $\mathbf{v}^*$. The model equation in hand is:

$$(1) \qquad \begin{cases} \mathbf{v}_t + (\mathbf{v}\cdot\nabla)\mathbf{v} - \mu\Delta\mathbf{v} = -\nabla p + \mathbf{f}, \quad \nabla\cdot\mathbf{v} = 0 \quad \text{in} \quad \Omega_t, \\ \mathbf{v}(\mathbf{x},0) = \mathbf{v}_0(\mathbf{x}) \quad \text{in} \quad \Omega_0, \\ \mathbf{v}(\mathbf{x},t) = \mathbf{v}^*(\mathbf{x}_{n-1},t) \quad \text{on} \quad \partial\Omega_t, \end{cases}$$

where Ω_t and $\partial\Omega_t$ denotes the fixed boundary if the domain is bounded. The solution $\mathbf{v}$ is the vector field representing the velocity of the flow in each direction, and its rotation $= \nabla\times\mathbf{v}$ is the vorticity. Note that $\nabla\cdot = 0$ in $\overline{\Omega}_t$ by compatibility. The solution is investigated in light of the most common classical definition of weak solutions in order to ensure the possibility of generalizing the subsequent results to the widest classes of possible solutions. In particular, we follow the definition of weak solutions introduced first by Leray [1–3]. Further, the investigation in this article establishes rigorously the previously investigated results in [4, 5].

Keywords: Fluid Mechanics, Navier-Stokes equation, Convection
2010 Mathematics Subject Classification: 76D03, 76D05, 76M30, 76R10

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Trigonometric system and optimal approximation

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Abstract: n-Widths were introduced to compare and classify the power of approximation of a wide range of algorithms. Optimality of the trigonometric system is a frequently discussed topic in the theory of n-widths [1,2]. We present a new phenomenon in the behavior of the trigonometric system in the the "usual" order, i.e. $\{1, \exp(\pm i\cdot), \exp(\pm 2i\cdot), \cdots\}$. Namely, the sequence of subspaces $\mathcal{T}_n$ of trigonometric polynomials of order n is optimal in the sense of order of Kolmogorov's n-widths $d_n(K * U_p, L_q)$ on convolution classes $K * U_p$ in L_q for all $1 < p, q < \infty$ in the case of "superhigh" smoothness (analytic and entire), that is in the case

$$K(x) = \sum_{k=1}^{\infty} \exp(-\mu k^{\varrho}) \cos kx, \mu > 0, \varrho \geq 1$$

[4,5] and "supersmall" smoothness, i.e. if

$$K(x) = \sum_{k=1}^{\infty} k^{-\left(\frac{1}{p}-\frac{1}{q}\right)_+} (\log k)^{-\nu} \cos kx, \nu > 0.$$

However, in the intermediate cases of "small smoothness" $(1/p - 1/q < r < 1/p)$ and "finite smoothness" [3] $(r > 1/p)$, i.e. if

$$K(x) = \sum_{k=1}^{\infty} k^{-r} \cos kx$$

the sequence of subspaces $\mathcal{T}_n$ is not optimal for all $1 < p, q < \infty$. Also, we present similar results for linear and Bernstein's n-width.

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Minisymposium: Modern Problems of Mathematical Hydrodynamics

Weak solvability of some fractional viscoelasticity models

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Abstract: The mathematical models of dynamics of viscoelastic fluids with constitutive relations of the form

$$\sum_{k=0}^{n}\sum_{i=0}^{s} b_{ki} D_{0t}^{k+\beta_{ki}} \sigma = \sum_{k=0}^{m}\sum_{i=0}^{r} a_{ki} D_{0t}^{k+\beta_{ki}} \varepsilon, \quad 0 \le \beta_k < 1, \qquad (l2)$$

where D_{0t}^{α} is the RiemannLiouville fractional derivative of order $\alpha > 0$, σ is the deviator of the stress tensor, and ε is the deformation tensor are under consideration. Fractional analogous of Voigt and anti-Zener models are under investigation. We establish the existence of weak solutions of the corresponding initial-boundary value problems. In the planar case the uniqueness of weak solutions is proved. For the proofs of the main results we approximate the problems under consideration by a sequence of regularized systems of Navier-Stokes type. The solvability of regularized systems and apriori estimates of their solutions allow to pass to the limit in the regularized systems and obtain the solvability of original problems. The theory of fractional powers of positive operators, fractional calculus and classical results on Navier-Stokes equations are used.

This is a joint work with Victor Zvyagin.

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Keywords: viscoelastic fluid; fractional derivative, Voigt model, anti-Zener model, weak solution, initial-boundary value problems.

2010 Mathematics Subject Classification: 35A01, 35Q35, 45K05, 76D03

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Investigation of the solvability of initial-boundary value problems for a viscoelastic model with memory

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This is join work with Prof. V.P. Orlov.

Abstract: Let $Q_T = [0,T] \times \Omega$, where $T > 0$, $\Omega \subset \mathbb{R}^n$, $n = 2,3$, is a bounded domain with boundary $\partial\Omega \subset C^2$. We consider in Q_T the problem

$$\frac{\partial v}{\partial t} + \sum_{i=1}^{n} v_i \frac{\partial v}{\partial x_i} - \mu_0 \Delta v - \mu_1 \mathrm{Div} \int_0^t e^{\frac{s-t}{\lambda}}\, \mathcal{E}(v)\,(s, z(s;t,x))\, ds + \nabla p = f; \tag{1}$$

$$z(\tau;t,x) = x + \int_t^\tau v\,(s, z(s;t,x))\; ds, \quad \operatorname{div} v(t,x) = 0; \tag{2}$$

$$v(0,x) = v_0(x),\ x \in \Omega; \quad v(t,x) = 0, \quad (t,x) \in [0,T] \times \partial\Omega. \tag{3}$$

Theorem 1 [1]. *Let $f = f_1 + f_2$, where $f_1 \in L_1(0,T;H)$, $f_2 \in L_2(0,T;V^{-1})$ and $v_0 \in H$. Then there exists a weak solution of problem* (1)-(3).

Let $Q = (-\infty, T] \times \Omega$, where $T > 0$, $\Omega \subset \mathbb{R}^n$, $n = 2,3$, is a bounded domain with boundary $\partial\Omega \subset C^2$. We consider in Q the problem

$$\frac{\partial v}{\partial t} + \sum_{i=1}^{n} v_i \frac{\partial v}{\partial x_i} - \mu_0 \Delta v - \mu_1 \mathrm{Div} \int_{-\infty}^t e^{\frac{s-t}{\lambda}}\, \mathcal{E}(v)(s, z(s;t,x)) ds + \nabla p = f; \tag{4}$$

$$\operatorname{div} v(t,x) = 0, \quad (t,x) \in Q; \quad v(t,x) = 0, \quad (t,x) \in (-\infty, T] \times \partial\Omega; \tag{5}$$

$$z(\tau;t,x) = x + \int_t^\tau v(s, z(s;t,x))\, ds, \quad t, \tau \in (-\infty, T], x \in \overline{\Omega}. \tag{6}$$

Theorem 2 [2]. *Let $f \in L_2(-\infty, T; V^{-1})$. Then problem (4)-(6) has at least one weak solution.*

Keywords: Weak solution, viscoelasticity, model with memory

2010 Mathematics Subject Classification: 35Q35, 76A10, 35D30, 35R09

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Trajectory and global attractors for termo-Voigt model

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Abstract: Let Ω be bounded domain in the space $\mathbb{R}^n$, $n = 2, 3$, with a smooth boundary $\partial\Omega$. We consider the following initial-boundary value problem

$$\frac{\partial v}{\partial t} + \sum_{i=1}^{n} v_i \frac{\partial v}{\partial x_i} - 2\mathrm{Div}(\nu(\theta)\mathcal{E}(v)) - \varkappa\frac{\partial \Delta v}{\partial t} + \nabla p = f; \tag{1}$$

$$\mathrm{div}\ v = 0; \qquad v|_{t=0} = v_0, \quad x \in \Omega; \qquad v|_{\partial\Omega\times[0,+\infty]} = 0; \tag{2}$$

$$\frac{\partial \theta}{\partial t} + \sum_{i=1}^{n} v_i \frac{\partial \theta}{\partial x_i} - \chi\Delta\,\theta = 2\nu(\theta)\mathcal{E}(v) : \mathcal{E}(v) + 2\varkappa\frac{\partial \mathcal{E}(v)}{\partial t} : \mathcal{E}(v) + g; \tag{3}$$

$$\theta|_{t=0} = \theta_0, \quad x \in \Omega; \qquad \theta|_{\partial\Omega\times[0,+\infty]} = 0. \tag{4}$$

Here, $v = (v_1(t,x), \dots, v_n(t,x))$ is un unknown vector-valued velocity function of particles in the fluid, $p = p(t,x)$ is un unknown pressure, $f = f(t,x)$ is the external force. The divergence $\mathrm{Div}\, C$ of the tensor $C = (c_{ij}(t,x))$ is the vector with with coordinates $(\mathrm{Div}\, C)_j = \sum_{i=1}^{n}(\partial c_{ij}/\partial x_i)$;

$$\mathcal{E}(v) = (\mathcal{E}_{ij}(v)), \quad \mathcal{E}_{ij}(v) = \frac{1}{2}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}\right), \quad i, j = \overline{1, n},$$

is the strain-rate tensor; $\nu > 0$ is the fluid viscosity, $\varkappa$ is the retardation time, v_0 and θ_0 are given functions.

Theorem 6.1. *The trajectory space $\mathcal{H}^+$ of problem* (1)-(4) *has the minimal trajectory attractor $\mathcal{U}$ and the global trajectory attractor $\mathcal{A}$.*

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Keywords: Non-Newtonian fluid, trajectory attractor, global attractor, existence theorem

2010 Mathematics Subject Classification: 35B41, 35Q35, 76D03

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Average magnetic field in a random stream with an update

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Abstract: In this paper we derive the equation of the average magnetic field in a random flow with an update and an asymptotic analysis of the obtained equation is carried out.

Consider the equation of magnetic induction $\overrightarrow{H} = \overrightarrow{H}(t,x)$ in a random flow $\overrightarrow{V}(t,x)$ of an incompressible medium with a constant magnetic viscosity ν_m $(t \geq 0, x \in R^3)$:

$$\frac{\partial \overrightarrow{H}}{\partial t} = \nu_m \triangle \overrightarrow{H} - (\overrightarrow{V}, \nabla)\overrightarrow{H} + (\overrightarrow{H}, \nabla)\overrightarrow{V}, \overrightarrow{H}(0,x) = \overrightarrow{H_0}(x). \tag{1}$$

Next, consider the problem (1) in the model of a random flow with an update: $\overrightarrow{V}(t,x) = \overrightarrow{V}_k(x), t \in [k\tau, (k+1)\tau), k = 0,1,2,..., \tau > 0$ is the update time, $\overrightarrow{V}_k(x)$ are independent identically distributed random velocity fields with nonzero means.

We denote by $\overrightarrow{\mathbf{H}}(t,x) = \langle \overrightarrow{H}(t,x) \rangle$ the average magnetic field in the distribution of the velocity field.

In [1], an equation for $\overrightarrow{\mathbf{H}}(t,x)$ was obtained by means of the transition to the Fourier transform $\overrightarrow{\mathbf{H}}(t,x)$ with respect to the spatial coordinate. In this paper, we derive the equation for the mean magnetic field directly, using the probabilistic representation of the solution (1) in the form of a conditional mathematical expectation along the trajectories determined by the operator $A = \nu_m \triangle - (\overrightarrow{V}, \nabla)$ of the random process. In the asymptotic analysis of the resulting equation, we use the expansions of the coefficients of the equation $\overrightarrow{\mathbf{H}}(t,x)$ with respect to the degree of the small update time τ.

Keywords: Random stream with update, a magnetic field, average magnetic field, conditional mathematical expectation.

2010 Mathematics Subject Classification: 60J25, 60J22, 60J55

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Minisymposium: Actual mathematical problems in control theory

Projector approach for constructing the zero order asymptotic solution for the singularly perturbed linear-quadratic control problem in a critical case

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Abstract: This report is devoted to linear-quadratic optimal control problems with a weak control, fixed left point and the state equation of the form

$$\varepsilon \frac{dx}{dt} = A(t,\varepsilon)x + \varepsilon B(t,\varepsilon)u + \varepsilon f(t,\varepsilon),\ t \in [0,T], \tag{1}$$

where $T > 0$ is fixed; $\varepsilon \geq 0$ is a small parameter; $x = x(t,\varepsilon)$ and $u = u(t,\varepsilon)$ are $m-$ dimensional and $r-$ dimensional vector functions, respectively; real-valued matrices of suitable size $A(t,\varepsilon)$, $B(t,\varepsilon)$, and $f(t,\varepsilon)$ are sufficiently smooth with respect to both parameters t and ε; moreover, the matrix $A(t,0)$ is singular.

Using the direct scheme method consisting in immediate substituting a postulated asymptotic solution into the problem condition and receiving problems for finding asymptotic terms, the zero order asymptotic solution is constructed under some conditions.

In contrast to paper [1], where the problem considered has been studied, the projector approach is applied here. This approach allows us to make the algorithm of constructing the asymptotic solution more clear and to correct some inaccuracies in [1].

Keywords: linear-quadratic optimal control problems, singular perturbations, weak controls, critical case, asymptotic expansions, boundary-layer functions, direct scheme method

2010 Mathematics Subject Classification: 49J15, 41A60, 34E15

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A Solution to Inverse Problems For Dynamic Control Systems [1]

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Abstract: Inverse problems of reconstruction of realized controls for dynamic control systems linear in controls and non-linear in phase coordinates are considered. It is assumed that inaccurate measurements of the realized trajectory are known.

Inverse problems have been studied by many authors. The approach suggested by Yu.S. Osipov and A.V. Kryazhimskii [1] is one of the closest to the presented material. It is originated form the differential games theory.

Another method for solving dynamic reconstruction problems by known history of inaccurate measurements has been suggested by N.N. Subbotina, E.A. Krupennikov and T.B. Tokmantsev [2, 3]. A modification of this approach is presented. It relies on necessary optimality conditions for an auxiliary variational problem of finding stationary points of a convexo-concave integral discrepancy functional. The functional is a variation of a Tikhonov regularizator.

Results of simulation are exposed.

Keywords: Nonlinear control systems, inverse problem, calculus of variations, hamiltonian system, necessary optimality conditions

2010 Mathematics Subject Classification: 49M05,34A55 **References:**

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A design of an approximate regulator for a weakly nonlinear terminal control problem using the penalty functions method

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Abstract: This report is devoted to the application of the penalty functions method to the construction of approximated terminal regulator for the next optimal control problem:

$$J(u) = \frac{1}{2}\int\limits_{t_0}^{t_1} \left(x^T Q(x,\varepsilon)x + u^T Ru\right) dt \to \min_u, \tag{1}$$

$$\dot{x} = A(x,\varepsilon)x + B(x,\varepsilon)u,\ x(t_0) = x^0,\ \ x(t_1) = 0,\ \ x \in X \subset R^n,\ u \in R^r,$$

where $0 < \varepsilon \leq \varepsilon_0 \ll 1, Q(x,\varepsilon) = Q_0 + \varepsilon Q_1(x) \geq 0, A(x,\varepsilon) = A_0 + \varepsilon A_1(x),\ B(x,\varepsilon) = B_0 + \varepsilon B_1(x)$. Thus, we consider the problem for a weakly nonlinear state-dependent coefficients (SDC) system. By means of introducing the quadratic penalty function

$$\frac{1}{2\mu} x^T(t_1)Fx(t_1),\ 0 < \mu \ll 1,\ F > 0,$$

problem (1) becomes a SDC optimal control problem with a free right-hand endpoint. It can be investigated on the basis of the approach proposed in [1]. First of all, using ε and μ matching, we study the asymptotics of the auxiliary equation solution. This equation is close to the differential matrix SDC Riccati equation obtained in [1]. Then, on the basis of asymptotics, an approximate terminal regulator with extrapolation properties is constructed.

This work was supported by the Russian Science Foundation (Project No. 17-11-01220).

Keywords: terminal control, nonlinear systems, small parameter, state-dependent Riccati equations, penalty method.

2010 Mathematics Subject Classification: 34E10, 34J15, 49M30, 49N35

Poster Presentations

A conjugate gradient method for global optimization via a random perturbations

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Abstract: In this paper a method for solving global optimization problems where the objective function is not necessarily convex is presented. This method is a modification of the Polak-Ribière conjugate gradient method involving the adjunction of a stochastic perturbation. This approach leads to a stochastic descent method where the deterministic sequence generated by Polak-Ribière conjugate gradient method is replaced by a sequence of random variables. Mathematical results concerning the convergence to a global minimum are established. Numerical experiments are achieved on some typical test problems, particularly the engineering problem of Lennard-Jones clusters. A comparison with well known methods is carried out to show the performance of our algorithm.

Keywords: Global optimization,Random perturbation, Polak-Ribière conjugate gradient method, Lennard-Jones clusters problem.

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Numerical simulation of magnetohydrodynamic process in metallurgical DC furnaces

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Abstract: In recent years metallurgical industry is search for modern technology based on low energy and resources consumption. One of modern approach for remelting of raw materials is DC furnaces with bottom electrodes. The exploitation of such furnaces has shown problems with increase wear of the bottom electrodes. The reason of this increase wear is vortex flow generated by electromagnetic forcing in liquid metal. In present work the model of preceding process of liquid metal in DC furnace with bottom electrodes, as well software based on the standard commercial software are developed [1]. This model provide possibility to carry out simulations of electric and magnetic parameters, temperature distribution with Joule heat and heat from the arc and hydrodynamic process with taking into account electromagnetic parameters as Lorentz force and heat exchange processes.

The work deals not only with development of the approaches for the numerical simulation but also advice methods for the control of liquid metal vortex flow in the DC furnace with the bottom electrodes by changes of bottom electrode parameters and furnace volume construction. It is shown that cooling down the bottom electrode to the melting metal temperature leads to the decrease of shear stress on the fettle area by 15 %, bottom electrode lifting above the surface at the electrode radius â“ by 30 %, and the bath form change â“ by 50 % [2].

Keywords: Numerical simulation, Electrovortex flow, DC metallurgical furnace

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Some approaches for numerical modeling of electrovortex flows in a finite volume

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Abstract: Electro Vortex flow is very important in various applications such as electrical metallurgical furnace, electrical welding, cristal growth, etc. This work presented investigation of electrovortex flows of a liquid metal in a finite volume for various electromagnetic and hydrodynamic regimes. The melt motion was described by magnetic hydrodynamic equations for an incompressible isothermal and non-isothermal fluid. A strategy for solving the problem with the use of standard packages of applied programs has been developed.

The laminar flow of the melt at small Reynolds numbers, for which there is a known solution, as well as the turbulent flow, for which there are experimental data, have been calculated. In calculating the turbulent flow, various turbulence models were used. It has been shown that the k-ϵ, k-ω and LES models describe satisfactorily the electrovortex flow of the liquid metal and agree well with experimental data. The good agreement between the results of calculations made with the use of various methods and software packages, as well as between theoretical and experimental data, provide the reliability of the developed methods and approaches and the validity of the obtained results.

Keywords: Lorentz force, turbulent electrovortex flow, numerical simulation

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Nonstationary Growth of Spherical Particles in a Supercooled Melt

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Abstract:

The processes of industrial production of many materials are frequently accompanied by phase transformations of the liquid-solid type. Such phase transitions often occur in the presence of a supercooled region - a two-phase zone. This zone is filled with crystals of solid phase, evolving in a metastable liquid. Depending on the supercooling, the crystals can be either microscopic aggregates or dendritic-like structures [1].

The evolution of an ensemble of microscopic particles in a supercooled melt is usually described using the kinetic equation for the particle-size distribution function and the balance equation for the temperature or supercooling of the liquid. The mathematical formulation of the problem depends on the law for the rate of growth of individual particles evolving in a supercooled medium. Traditionally, the steady-state approximation is used for this velocity, which follows from the solution of the stationary Stefan thermal problem for a spherical particle [2]. However, the release of heat on the surfaces of growing crystals significantly changes the temperature field around them and leads to the need to consider the non-stationary process of their growth. In this paper, the main nonstationary corrections to the steady-state law for the growth rate of spherical crystals in a supercooled metastable liquid are determined.

Keywords: Moving boundary problem, partial differential equations, Stefan problem

2010 Mathematics Subject Classification: 82C26, 35R37, 80A20

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On the Theory of Two-Phase Zone Incipience due to the Effect of Constitutional Supercooling

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Abstract:

Directional crystallization of binary melts is one of the ways of obtaining solid materials, the quality of which is completely determined by the physical and regime parameters of the solidification process. It is well known that, under certain conditions, the solid-liquid interface becomes morphologically unstable [1] and the constitutional supercooling occurs ahead of the planar solidification front [2], which leads to the appearance of a metastable region. In this supercooled region, crystals can grow in the form of dendrites, grow on impurity inclusions, etc. As this zone is transitional between the already formed crystal and the melt, it is called a two-phase zone.

In this paper, we present the results of numerical experiments carried out in accordance with the model taking into account the formation of a two-phase zone. In particular, qualitative and quantitative patterns of the time dependence of the formation of a two-phase zone in the Fe-Ni melt are obtained under two different regimes of active and passive cooling. Based on the results of calculations, it can also be concluded that, in both cases, there are such values of the cooling parameters, under which a two-phase zone is formed very close to the right boundary of the sample, i.e. practically the whole sample remains homogeneous. Choosing the appropriate cooling mode and its parameters makes it possible to optimize the metallurgical process both in the quality of the obtained alloys and in the speed of their production.

Keywords: Moving boundary problem, partial differential equations, Stefan problem

2010 Mathematics Subject Classification: 82C26, 35R37, 80A20

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Analytical Description of Directional Crystallization with Two-Phase Regions

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Abstract:

The present paper is devoted to the construction of an analytical solution of a nonlinear nonstationary directional crystallization model with two moving boundaries. The nonstationary process of directional crystallization of three-component melts (solutions) is investigated in the presence of two phase transition zones located between the pure solid and liquid phases. An analytical solution of the nonlinear system of heat and mass transfer equations is found in the presence of two moving interphase boundaries. The temperature distributions, the concentrations of dissolved impurities, and the fractions of the solid and liquid phases in the two-phase zones are determined. The laws of motion of the boundaries between two-phase zones are found too. We consider the heat and mass transfer equations in the moving two-phase zone

$$\rho c \frac{\partial T_p}{\partial t} = \frac{\partial}{\partial z}\left(k(\chi)\frac{\partial T_p}{\partial z}\right) + L_V \frac{\partial \varphi_A}{\partial t}, \tag{1}$$

$$\chi \frac{\partial B}{\partial t} = D_B \frac{\partial}{\partial z}\left(\chi \frac{\partial B}{\partial z}\right) + B\frac{\partial \varphi_A}{\partial t},\ \chi \frac{\partial C}{\partial t} = D_C \frac{\partial}{\partial z}\left(\chi \frac{\partial C}{\partial z}\right) + C\frac{\partial \varphi_A}{\partial t}, \tag{2}$$

where T_p is the temperature, B and C are the concentration of impurity, $k = k_l\chi + k_s(1-\chi)$, ρ, c, k_l, k_s, D_B and D_C represent the constant coefficients. The solid phase fractions φ_A, φ_B and the liquid phase fraction χ are the time-dependent functions of the spatial coordinate z. This problem has been solved on the basis of techniques developed in our previous works [1,2].

Keywords: Moving boundary problem, partial differential equations, self-similar variables

2010 Mathematics Subject Classification: 82C26, 35R37, 80A20

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Nonlinear Dynamics of Unsteady-State Crystallization of a Binary System

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Abstract:

Quite often, when the materials solidify between a purely liquid and a purely solid phase, a zone of a two-phase state of matter (liquid and solid) is formed. Its formation is a reaction of the system to the onset of instability of a planar front and the appearance of constitutional supercooling. Equations describing the processes of heat and mass transfer in a two-phase zone are complex nonlinear partial differential equations. The boundary conditions for these equations should be formulated at two moving boundaries, the positions of which are unknown. Thus, the problem of determining the fields of temperature and concentration in a two-phase zone, its dimensions and position, is a complex nonlinear system of thermophysical equations with moving boundaries [1,2].

The present paper is concerned with analytical description of self-similar crystallization regime in the presence of a phase transition layer (two-phase zone) filled with the growing solid phase and liquid. A nonlinear heat and mass transfer model is simplified to describe the case of small variations of the solid phase fraction in the two-phase zone. An exact analytical solution of simplified model is obtained. We show that this solution is in good agreement with the previously known theory. An important point is that the obtained solution has a wider scope of applicability than the asymptotic solution previously found in the vicinity of the bifurcation point.

Keywords: Moving boundary problem, partial differential equations, self-similar variables, two-phase layer

2010 Mathematics Subject Classification: 82C26, 35R37, 80A20

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How the convective heat transport at the solid/liquid phase interface influences the stable mode of dendritic growth

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Abstract:

Convection plays an essential role in the growth of dendrites. Convection may influence the transport of heat and substances as well as mechanical deformation of dendritic crystals. In this paper, the effect of convective heat transport that substantially changes the dendrite tip diameter and its tip velocity is demonstrated. In addition, the Gibbs-Thomson condition connecting the phase transition temperature has been taken into account with allowance for the kinetic contribution arising from the effect of attachment kinetics at the phase transition boundary [1].

In this paper the growth of an anisotropic dendrite under conditions of convective heat transfer at the solid-liquid interface was analyzed theoretically. An analytical solution is obtained for the temperature distribution in the liquid phase for the parabolic and paraboloidal forms of the needle-like dendrite. The stability analysis is carried out and the integral of microscopic solvability is defined [2]. The selective ratio for the stable growth rate of the dendritic tip and its diameter is derived.

Keywords: Dendritic growth, Phase transitions, Solvability condition, Convective heat and mass transfer

2010 Mathematics Subject Classification: 82C26, 35R37, 80A20

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Stability anlaysis of conformable fractional order nonlinear systems

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Abstract: In this paper, we study the uniformly fractional exponential stability for some class of systems like class of perturbed systems and class of nonlinear fractional-order equations with control using the Lyapunov-like function.

Keywords: Conformable fractional derivative, fractional practical exponential stability.

A new sufficient conditions on the generalized spectrum method to deal with spectral pollution

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Abstract: In this work we solve the spectral pollution. We suggest a modern method based on generalized spectral techniques, where we show that the propriety L is hold with norm convergence. We describe the theoretical foundations of the method in details, as well as we illustrate its effectiveness by numerical results.

It is well known that the spectral pollution is the weakness of projection methods for an unbounded operator. Thus, our technic is an alternative method based on disrupting an unbounded operator by a bounded one until that the spectral properties transform on controlled case. The generalized spectrum takes its place as the desired case. We will show that every unbounded operator contains a decomposition of two bounded operators which carry all the spectral properties. Through this decomposition and basing upon its numerical approximations the phenomenon of spectral pollution will be resolved.

The natural framework of our research is a complex separable Banach space $(X, ||.||)$. Our numerical results are applied on harmonic oscillator operator, which is defined over $L^2(\mathbb{R})$ by

$$Au = -u'' + x^2 u. \tag{1}$$

Generalized spectrum. Let T and S two operators in $\mathrm{BL}(X)$, we define the generalized resolvent set by

$$\mathrm{re}(T,S) = \{\lambda \in \mathbb{C} : (T - \lambda S) \text{ is bijective}\}. \tag{2}$$

The generalized spectrum set is $\mathrm{Sp}(T,S) = \mathbb{C} \setminus \mathrm{re}(T,S)$. For $z \in \mathrm{re}(T,S)$, we define $R(z,T,S) = (T - zS)^{-1}$, the generalized resolvent operator.

We define $\lambda \in \mathbb{C}$ as a generalized eigenvalue when $(T - \lambda S)$ is not injective, then the set $E(\lambda) = \mathrm{Ker}(T - \lambda S)$ is the generalized spectral subspace. We say that λ has an finite algebraic multiplicity if there exist α where $\dim \mathrm{Ker}(T - \lambda S)^\alpha < \infty$. If the operator S is invertible, we have $\mathrm{Sp}(T,S) = \mathrm{Sp}(S^{-1}T)$.

Theorem 8.1. *Let $\lambda \in re(T,S)$ and $\mu \in \mathbb{C}$ where $|\lambda - \mu| < ||R(\lambda,T,S)S||^{-1}$, then $\mu \in re(T,S)$.*

Theorem 8.2. *The set $Sp(T,S)$ is closed in $\mathbb{C}$.*

Theorem 8.3. *The function $R(\cdot,T,S) : re(T,S) \to BL(X)$ is analytic, and its derivative given by $R(\cdot,T,S)SR(\cdot,T,S)$.*

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On the Stability of High-Performance Crystal Growth Simulations by the MPFC Method

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Abstract: Computer modeling of crystal growth processes is often associated with complex numerical calculations. The results of such calculations cannot always be predicted. In some cases, the features of mathematical methods of calculation in conjunction with the assumptions of mathematical models can lead to the appearance of unexpected results. Such results on the one hand will be correct from a mathematical point of view, but on the other hand they will not have physical justification and experimental confirmation. Obviously, in such cases it is necessary to carefully process the results obtained taking into account the physical meaning and to avoid false conclusions. And it is especially important to take this into account in the case of large-scale high-performance simulations. Earlier, high-perfomance computational algorithm for the modified phase field crystal model (MPFC) of crystal growth were formulated [1] and the results of some three-dimensional simulations are obtained [2]. In current paper we present some unstable results of simulations by the MPFC and discuss such a phenomenon.

Keywords: Mathematical modeling, simulations, crystal growth, phase field crystal

2010 Mathematics Subject Classification: 35Q35,35Q86

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Algorithm of solution of a linear matrix inhomogeneous difference equation with small step

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Abstract: There is an algorithm for solving a linear matrix inhomogeneous difference equation is proposed in this paper. Also the conjugacy conditions of the homogeneous matrix difference equation and the ad joint equation to this equation are proved.

Keywords: A singularly perturbed differential equation, a system with a small parameter, a simple structure matrix, multiple small steps.

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About one inverse problem of time fractional evolution with an involution perturbation

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Abstract: We consider an inverse problem for a one-dimensional fractional evolution equation with involution and with periodic boundary conditions with respect to a space variable.

We will consider a process which is so slow that it is described by an evolutionary equation with a fractional time derivative. Thus, this process is described by equation

$$D_*^\alpha \Phi(x,t) - \Phi_{xx}(x,t) + \varepsilon \Phi_{xx}(-x,t) = f(x) \tag{1}$$

in the domain $\Omega = \{(x,t):\ -\pi < x < \pi,\ 0 < t < T\}$. Here $f(x)$ is the influence of an external source that does not change with time; $t = 0$ is an initial time point and $t = T$ is a final one; and the derivative D_*^α defined in [1] as

$$D_*^\alpha u(x,t) = D_{0t}^\alpha \{u(x,t) - u(x,0) - t u_t(x,0)\},\ 1 < \alpha < 2$$

is a Caputo derivative for a regular function built on the Riemann-Liouville derivative D_{0t}^α. Such a Caputo derivative allows us to impose initial conditions in a natural way.

As the additional information we take values of two initial and one final conditions of the temperature

$$\Phi(x,0) = \phi(x),\quad \Phi_t(x,0) = \rho(x),\quad \Phi(x,T) = \psi(x),\quad x \in [-\pi,\pi]. \tag{2}$$

Since the wire is closed, it is natural to assume that the temperature at the ends of the wire is the same at all times:

$$\Phi(-\pi,t) = \Phi(\pi,t),\ t \in [0,T]. \tag{3}$$

Consider a process in which the temperature at one end at every time point t is proportional to the (fractional) rate of change speed of the average value of the temperature throughout the wire. Then,

$$\Phi(-\pi,t) = \gamma D_*^\alpha \int_{-\pi}^{\pi} \Phi(\xi,t)\, d\xi,\ t \in [0,T]. \tag{4}$$

Here γ is a proportionality coefficient.

Thus the investigated process is reduced to the following mathematical inverse problem: *Find a right-hand side $f(x)$ of the subdiffusion equation (1), and its solution $\Phi(x,t)$ subject to the initial and final conditions (2), the boundary condition (3), and condition (4).*

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Keywords: Inverse problem; fractional evolution equation; equation with involu- tion; periodic boundary conditions; method of separation of variables.

2010 Mathematics Subject Classification: 35K20, 35L15, 35R11, 35R30

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Canonical transformation and feedback linearization of state-dependent coefficient nonlinear systems

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Abstract: This report is devoted to a feedback linearisation problem for a nonlinear system with state-dependent coefficients:

$$\dot{x}(t) = A(x)x + B(x)u,\ x(0) = x_0, \tag{1}$$
$$x \in R^n,\ u \in R^1.$$

Here $A(x), B(x)$ are state-dependent matrices of the dimensions $n \times n, n \times 1$ correspondingly.

The problem is to find a non-singular similarity transformation $z = T(x)x$ such that the system (1) can be transformed to a quasi-linear canonical form

$$\dot{z}(t) = A_c(z)z + B_c(z)u,\ z(0) = z_0, \tag{2}$$

where $A_c(z),\ B_c(z)$ is a canonical pair in the form of Brunovskiy. This canonical transformation significantly simplify feedback linearization problem for transformed system (2), and knowing the inverse transformation $x = T^{-1}(z)z$, one can obtain a feedback linearization control for original system (1).

There are several techniques for designing such canonical transformation [1,2]. In this paper we introduce another approach based on the concept of state-dependent transfer function and controllability matrix for nonlinear system (1).

Keywords: state-dependent coefficient, nonlinear system, feedback linearisation, canonical transformation

2010 Mathematics Subject Classification: 35J05, 35J08, 35J25

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Strong approximation by Fourier series and generalized periodic Nikol'skij-Besov-Morrey spaces

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Abstract: In recent years, a lot of attention has been paid to study of Morrey type spaces. Many applications in partial differential equation of Morrey spaces and Nikol'skij-Besov spaces have been given in work G. Di Fazioand, M. Ragua and the book of T. Mizuhara. The theory of generalized Nikol'skij-Besov-Morrey spaces is developed. Generalized Morrey spaces, with T. Mizuhara and E. Nakai proposed, are equipped with a parameter and a function. In the present paper we study Nikol'skij-Besov spaces on scale generalized periodic Morrey space. Finally, we get an interesting characterization of the space $\mathcal{N}^s_{\varphi,p,q}(\mathbb{T})$ by using differences of partial sums of the Fourier series and consequence for strong approximation of Fourier series on Morrey space.

2010 Mathematics Subject Classification: 35J05, 35J08, 35J25

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Fourier method approach in mixed problems for the heat equation with involution perturbation

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Abstract: In this paper we consider a mixed problem for the parabolic equation with involution of the form

$$u_t\left(x,t\right)=u_{xx}\left(x,t\right)-\alpha u_{xx}\left(-x,t\right)-q\left(x\right)u\left(x,t\right), \quad -1<x<1, \quad t>0; \qquad (1)$$

$$u\left(0,x\right)=\varphi\left(x\right), \quad -1\leq x\leq 1; \quad u\left(-1,t\right)=0, \quad u\left(1,t\right)=0, \quad t\geq 0. \qquad (2)$$

In order to solve the problem we apply the separation of variables supposing $u(x,t)$ is a product:

$$u\left(x,t\right)=X\left(x\right)T\left(t\right).$$

Here the function $X\left(x\right)$ gives the solution to the boundary value problem

$$-X''\left(x\right)+\alpha X''\left(-x\right)+q\left(x\right)X\left(x\right)=\lambda X\left(x\right), \quad -1<x<1, \quad |\alpha|<1,$$
$$X\left(-1\right)=0, \quad X\left(1\right)=0. \qquad (3)$$

Theorem 1. If the number $\sqrt{\frac{1-\alpha}{1+\alpha}}$ is not even then the system of eigenfunctions of the spectral problem (3) forms the basis in $L_2(-1,1)$.

Theorem 2. Let the following conditions be satisfied:

1) a real continuous function $q\left(x\right)$ is non-negative;

2) the number $\sqrt{\frac{1-\alpha}{1+\alpha}}$ is not even $(-1<\alpha<1)$;

3) the initial condition $\varphi\left(x\right)$ of the mixed problem (1), (2) satisfies the conditions $\varphi\left(-1right\right)=\varphi\left(1\ \right)=0\,,$

Then the solution to the mixed problem (1), (2) exists, is unique and can be represented by the series

$$u\left(x,t\right)=\sum_{k=1}^{\infty}A_k e^{-\lambda_k t}X_k\left(x\right).$$

This work was supported by the Science Committee of the Ministry of Education and Science of the Republic of Kazakhstan, grant AP05131225.

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Modeling of quadratic programming of the mutual influence of elements of a dynamic series on examples of small agricultural formations

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Abstract: Proposed model effectively takes into account the mutual influence of the elements of the dynamic series that influence the increase of competitiveness of small agricultural formations, that is, the influence of various economic parameters on each other when they simultaneously manifest themselves. In this case, the forecasting operator is actually trained on the statistical material of the past. From this point of view, the prediction model introduced by us is a neural network.

Keywords: Quadratic Programming, Dynamic Series, Forecasting Operator

2010 Mathematics Subject Classification: 90C20, 90C39

Development of the method and calculation of the heat transfer coefficient of the soil

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Abstract: In this work an approximate method for determining the coefficients of heat transfer of soil is developed. The limitation of approximate value of the heat transfer coefficient and the monotony of the functional are proved. Algorithms for calculating the heat transfer coefficients of soil are developed and software product is designed. Using the measured data the heat transfer coefficient of rock is determined in the different areas of the open mine Ekibastuzsky. Nowadays one of the most relevant problems is to search and elaborate energy saving actions on creation of heat and technological processes with minimum thermal losses. Prior knowledge of thermo physical characteristics of the used and developed structural, heat insulating and lining materials has a huge impact. Thermal characteristics of enclosing constructions significantly influence the thermal and air conditions of the building and the work of heating system. Problems of energy saving and decrease in heat losses significantly influences an ecological situation, technical and economic indicators and capital expenditures of structural objects. In order to solve these problems it is necessary to know thermal capacity, heat transfer, heat conductivity and heat diffusivity of materials. Some materials have passport data, but others do not. Besides actual characteristics of structural materials can change during the operation and do not to correspond to their certificate or the passport. Therefore during the construction of objects of different function it is necessary to be able to define thermal characteristics of materials.

A very effective characteristic of the heat exchange process is the heat transfer coefficient h, between the air flow and the surface of soil. A lot of researches reveals the effect on the given dependence of the porosity of the material, the size of a form of the grains, and the effect of all possible physical characteristics of the material (for example, heat capacity, thermal conductivity). There is a huge amount of empirical formulas. But both the data and the formulas that form the basis for them differ sharply at various authors. There are a lot of reasons of a divergence: the influence of the internal resistance of the pieces; Complex geometric structure of the layer, which changes from various factors; the porosity of the layer, and the roughness of the grains. The value of the heat transfer coefficient is strongly dependent on the amount, thickness and thermophysical characteristics of soil layers. In the depth of the cut, values of the heat transfer coefficient decreases. The coefficient has a significant difference at wind speed.

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On the Well-Posedness of the Nonlocal Boundary Value Problem for the Differential Equation of Elliptic Type

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Abstract: In the present paper the abstract nonlocal boundary value problem for the second order differential equation

$$-v''(t) + Av(t) = f(t) \quad (0 \leq t \leq T), v(0) = v(T) + \varphi, \int\limits_0^T v(s)ds = \psi$$

in an arbitrary Banach space E with the positive operator A is considered. The well-posedness of this problem in various Banach spaces is established. In applications, the coercive stability estimates in Hölder norms for the solutions of the mixed type nonlocal boundary value problems for elliptic equations are obtained.

Keywords: elliptic equation; positive operators; Well-posedness; coercive stability.

2010 Mathematics Subject Classification: 35J25, 47E05, 34B27

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Error a posteriori estimation for a problem of optimal control of the obstacle

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Abstract: This poster is devoted to a problem of the optimal control of the obstacle represented as follows

$$\min_{\psi\in U_{ad}} \left\{ \frac{1}{2}\int_\Omega (\mathcal{T}_g(\psi)-z)^2 dx + \int_\Omega (\nabla\psi)^2 dx \right\} \tag{1}$$

subject to:

$$\forall v \in \mathcal{K}(\psi):\; a(y,\, v-y) + \langle g(y)-f,\, v-y\rangle \geq 0\,;\; y\in\mathcal{K}(\psi) \tag{2}$$

such that

$$\mathcal{K}(\psi) = \left\{ y\in H_0^1(\Omega);\, y\geq\psi, \;\; \text{pp}\,\text{dans}\,\Omega \right\} \tag{3}$$

$$a(u,v) = \sum_{i=1}^n \left(\sum_{j=1}^n a_{ij} \int_\Omega \frac{\partial u}{\partial x_i}\frac{\partial v}{\partial x_j} dx + b_i \int_\Omega v \frac{\partial u}{\partial x_i} dx \right) + \int_\Omega cuvdx \tag{4}$$

where $a_{i,j}$, b_i and c in $L^\infty(\Omega)$. Moreover, on suppose $a_{i,j}\in\mathcal{C}^{0,1}(\Omega)$ (space of Lipschitz functions continues on Ω) and c non negative, where the form bilinear $a(.,.)$ is continuous and coercive with M et m are respective

Despite the non-differentiability of the $y=\mathcal{T}_g(\varphi)$ application, this type of problem has been widely studied by several authors on a mathematical level (see for example [1,2] but little has been done digitally.

After finite element discretization, and in spite of the non-convex character of this kind of problem, our main goal is to give estimates of the a posterior errors on the state and the control (see for example [2]).

Keywords: Probleme of the obstacle, optimal Control , Penalization

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Time asymptotic behavior of exponential function of Sturm-Liouville operator on the line

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Abstract: Spectral singularities of Sturm-Liouville operator on the line generate some increasing composants in time asymptotic of the solution of corresponding evolution equation. The calculus of these composants is given using Friedrichs' model of Sturm-Liouville operator and some scalar function which characterize the point of discontinuity of Fourier transformation of Friedrichs' model. An example is given.[1]

Keywords: Spectral singularities, Sturm-Liouville operator, Weyl function, Friedrichs' model, point spectrum, asymptotic behavior.

AMS subject classification:47A20, 47A30, 47H12, 46B34

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On one class of problems for restoring the density of sources of the process of fractional heat conductivity with respect to initial and final temperature

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Abstract: We consider a class of problems modeling the process of determining the temperature and density of sources in the process of fractional heat conductivity with respect to given initial and finite temperatures.

Let $\Omega = \{(x,t)\,,\ 0 < x < 1,\ 0 < t < T\}$. In Ω consider a problem of finding the right-hand side $f(x)$ of the fractional heat equation

$$D^{\alpha}_{0+}\left(u\left(x,\ t\right)\ -\ u\left(x,\ 0\right)\right) - u_{xx}\left(x,\ t\right) = f\left(x\right),\ \left(x,\ t\right) \in \Omega \tag{1}$$

and its solutions $u\left(x,t\right)$ satisfying the initial and final conditions

$$u\left(x,0\right) = \varphi\left(x\right),\quad u\left(x,T\right) = \psi\left(x\right),\quad 0 \le x \le 1, \tag{2}$$

and the boundary conditions

$$\left\{\begin{array}{l} a_1 u_x\left(0,t\right) + b_1 u_x\left(1,t\right) + a_0 u\left(0,t\right) + b_0 u\left(1,t\right) = 0, \\ c_1 u_x\left(0,t\right) + d_1 u_x\left(1,t\right) + c_0 u\left(0,t\right) + d_0 u\left(1,t\right) = 0. \end{array}\right. \tag{3}$$

The coefficients $a_k,\ b_k,\ c_k,\ d_k$ with $k = 0,1$ in (3) are real numbers, D^{α}_{0+} stands for the Riemann-Liouville fractional derivative of order $0 < \alpha < 1$:

$$D^{\alpha}_{0+}y\left(t\right) = \frac{1}{\Gamma(1-\alpha)}\frac{d}{dt}\int_0^t \frac{y(s)ds}{(t-s)^{\alpha}},$$

while $\varphi\left(x\right)$ and $\psi\left(x\right)$ are given functions.

Problems with general boundary conditions with respect to a spatial variable that are not strongly regular are investigated. The problem is considered independently of whether a corresponding spectral problem for an operator of multiple differentiation with not strongly regular boundary conditions has the basis property of root functions. The existence and uniqueness of classical solutions to the problem are proved.

The closest to the subject of this paper is [1], in which one case of regular but not strongly regular boundary conditions was considered.

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Keywords: Inverse problem; heat equation; fractional heat conductivity; not strongly regular boundary conditions; method of separation of variables

2010 Mathematics Subject Classification: 35K20, 35R30, 35R10

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Improving Performance of Indoor Visible Light Communication Systems with Hexagonal Designed LED Transmitters

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Abstract: We report a simulation program for indoor visible light communication with hexagonal LED array shape that will allow us to focus the luminance of optical light for the room center with neglecting of the boundary since they will be occupied by furniture. Users will not be in those areas. We are going to model OOK, PPM and QAM modulation techniques with varied semi-angle degrees, as a result we want to prove with hexagonal shape we could make and uniform luminance in the room. We are going to use MATLAB and Simulink for simulation.

In this indoor VLC system both LOS and diffuse configurations will be taken into consideration. So, when received optical power at a point calculated, direct path and reflections from the wall should be taken into account as below [1]

$$P_r = \sum^{N_{LEDs}} P_t H_d(0) + \int_{reflections} P_t dH_{ref}(0)$$

The horizontal intensity at a point *(x,y,z)* is given as

$$I_{hor} = \frac{I(0).cos(\phi)^m}{D_d^2.cos\psi}$$

Keywords: Visible light communications (VLC), LED, illumination, modulation.

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Approximation of Maxwell's microscopic boundary conditions for onedimensional nonstationary nonlinear Boltzmann's twelve moment equations

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An important problem in aerospace engineering is the prediction of aerodynamic characteristics of aircraft at very high speeds and at high altitudes. In the case of a gas flowing near a solid or inside a region bounded by a solid surface, the boundary conditions describe the interaction of gas molecules with solid walls. The interaction of gas with a solid surface is the source of the drag and the lifting force of the body in the gas flow, as well as the heat transfer between the gas and the solid boundary. The boundary conditions that the particle distribution function must satisfy at the boundary of the region where the particles under study are moving depend on the state of the boundary surface, its temperature, and on the degree of its roughness and purity. Unfortunately, theoretical and experimental studies on the interactions of gas with the surface have been little carried out. The mirror-diffusive Maxwell model [1] is often used as the boundary condition imposed on the distribution function of the gas molecules reflected from the surface of the molecules. To calculate the aerodynamic characteristics of aircraft, the Boltzmann's moment equations are used. Finite system of moment equations for a specific task with a certain degree of accuracy replaces the Boltzmann equation. It's necessary, also roughly, to replace the boundary conditions for the particle distribution function by a number of macroscopic conditions for the moments, i.e. there arises the problem of boundary conditions for a finite system of equations that approximate the microscopic boundary conditions for the Boltzmann equation. The question of boundary conditions for a finite system of moment equations can be divided into two parts: how many conditions must be imposed and how they should be prepared. From microscopic boundary conditions for the Boltzmann equation there can be obtained an infinite set of boundary conditions for each type of decomposition. However, the number of boundary conditions is determined not by the number of moment equations, i.e. it is impossible, for example, take as much boundary conditions as equations, although the number of moment equations affects the number of boundary conditions. In addition, the boundary conditions must be consistent with the moment equations and the resulting problem must be correct. The initial and boundary value problem for one-dimensional non-stationary Boltzmann's equation with boundary conditions of Maxwell was approximated by a corresponding problem for the Boltzmann's moment system of equations [2]. In this work we approximate the microscopic Maxwell boundary condition satisfied by the particle distribution function for one dimensional Boltzmann's twelve moment equations and show correctness of the initial and boundary value problem for Boltzmann's twelve moment system equations.

Keywords: Boltzmann equation, boundary conditions, nonlinear, Boltzmann's moment system equations, particle distribution functions.

2010 Mathematics Subject Classification: 35Q20, 35F60

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The modeling of heat and mass transfer processes in colloidal suspension

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Abstract: In the current paper, we consider a liquid-phase medium with nanoparticles irradiated by a light beam with a Gaussian intensity profile. As a result of the action of the light field in the medium, gradients of temperature and concentration arise, which cause heat and mass transfer. These phenomena are described by a system of balance equations for temperature and particles both with concentration convection. Let us take into account the fact that the temperature establishment processes go faster than the diffusion processes. Therefore, we study the diffusion processes at a stationary temperature. As a result of introducing dimensionless variables and parametrization, we obtained an initial boundary value problem for a one-dimensional nonlinear parabolic equation.

In this case, the intensity with a Gaussian profile was considered as the initial condition. The boundary conditions are obtained by equating to zero the sum of the thermodiffusion and electrostriction flows of nanoparticles. In deriving the parabolic equation, we used a linear dependence of the thermal conductivity of the medium on temperature. Such dependence is confirmed by a number of experiments. We use the predictor-corrector method for the numerical solving of the nonlinear initial boundary value problem. The constructed difference scheme is unconditionally stable and has second-order accuracy in time and space variables. In this paper, we presented the conclusions about the dynamics of concentration using the numerical experiment results.

Throughout this note we mainly use techniques from the paper [1].

Keywords: heat and mass transfer, dynamics of nanoparticles, Sore effect, non-linear equation, predictor-corrector method

2010 Mathematics Subject Classification: 35K05, 35K55, 35K57

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On a parabolic equation with time delay

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Abstract: In this study, exact solution of a delay parabolic equation is found by using separation of variables and Fourier series method [3].We find the numerical solution of initial-boundary value problem for delay parabolic differential equations with Neumann condition by using difference schemes [1-2]. Numerical solutions found by first and second order of accuracy difference schemes are given with tables.

Keywords: Delay parabolic equation, difference schemes

2010 Mathematics Subject Classification: 65M06, 35K10

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HAM Solution for a delay parabolic equation

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Abstract: In this study, exact solution of initial-boundary value problem for a delay parabolic differential equation with Neumann condition is found by using Laplace transform method. By using Homotopy Analysis Method (HAM), we find the numerical solution for this initial-boundary value problem [1]. Comparison of numerical solutions obtained by using HAM and difference schemes is presented in a table [2].

Keywords: Parabolic differential equation, time delay, homotopy analysis method, Laplace transform

2010 Mathematics Subject Classification: 35K10, 65M06, 65M99

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The construction of a solution of a related system of the Laguerre type

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Abstract: The systems of the Laguerre type, which were obtained from the system of Horn by means of exponential transformation, are considered. The Frobenius-Latysheva method is applied for the construction of the solution. The main theorem on the existence of four linearly independent particular solutions, which are expressed in terms of the degenerate hypergeometric function of M.R. Humbert $\Psi_2(\alpha,\gamma,\gamma';x,y)$ in the form of normally-regular series dependent on Laguerre polynomials of two variables, is proved.

Formulation of the problem

From the system of Horn

$$\left.\begin{aligned} &x\cdot Z_{xx}+(\gamma-x)\cdot Z_x-y\cdot Z_y-n\cdot Z=0,\\ &y\cdot Z_{yy}+(\gamma'-y)\cdot Z_y-x\cdot Z_x-n\cdot Z=0, \end{aligned}\right\} \tag{1}$$

by means of converting a system of Laguerre type is installed

$$\left.\begin{aligned} &x^2\cdot U_{xx}-xy\cdot U_y+\left(-\frac{x^2}{4}-\frac{xy}{2}+kx+\frac{1}{4}-\alpha^2\right)\cdot U=0,\\ &y^2\cdot U_{yy}-xy\cdot U_x+\left(-\frac{y^2}{4}-\frac{xy}{2}+kx+\frac{1}{4}-\beta^2\right)\cdot U=0, \end{aligned}\right\}. \tag{2}$$

where $k=(\alpha+\beta+2-2\lambda)/2$ is related with the basic Laguerre system [1]. Theorem is proved.

Theorem. The system (2) has four linearly independent partial solutions, which are expressed through the degenerate hypergeometric function of M.R. Humbert $\Psi_2(\alpha,\gamma,\gamma';x,y)$ in the form of normal-regular series

$$\begin{aligned} U(x,y)&=\exp\left(-\frac{x}{2}-\frac{y}{2}\right)\cdot x^{\frac{\alpha+1}{2}}\cdot y^{\frac{\beta+1}{2}}\cdot\Psi_2(-n,\alpha+1,\beta+1;x,y)=\\ &=\exp\left(-\frac{x}{2}-\frac{y}{2}\right)\cdot x^{\frac{\alpha+1}{2}}\cdot y^{\frac{\beta+1}{2}}\cdot L_{n,n}^{(\alpha,\beta)}(x,y) \end{aligned} \tag{3}$$

dependent on the Laguerre's polynomial of two variables

$$L_{n,n}^{(\alpha,\beta)}(x,y)=\Psi_2(-n,\alpha+1,\beta+1;x,y).$$

Frobenius-Latysheva method is used to proof of the theorem . Series of theorems on the necessary condition for the existence of a solution of the form are also proved (3).

Keywords: Related system, the system of the Laguerre type, the system of Horn, normal-regular solution, special curves, rank, anti rank.

2010 Mathematics Subject Classification: 34K29, 60H10

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Construction of solutions of inhomogeneous systems of Jacobi type

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Abstract: The possibilities of constructing a general solution of an inhomogeneous system of second-order partial differential equations of Jacobi type, closest to ordinary differential equations of the second order are studied. The method of undetermined coefficients is used for the construction of a particular solution of the system. Specific examples established a connection between systems of Jacobi type with one partial differential equation of the second order obtained by adding two equations of the original system.

Formulation of the problem

The method of undetermined coefficients is spread for the construction of a general solution of the inhomogeneous system of Jacobi type

$$\begin{aligned} &x\cdot(1-x)\cdot Z_{xx}+[\gamma-(\alpha+1)\,x]\cdot Z_x+n(\alpha+n)\cdot Z=f_1(x,y),\\ &y\cdot(1-y)\cdot Z_{yy}+[\gamma'-(\beta+1)\,y]\cdot Z_y+m(\alpha+m)\cdot Z=f_2(x,y),\end{aligned} \tag{1}$$

where $Z=Z(x,y)$ – the general unknown, $f_l(x,y)$ $(l=1,2)$ – polynomials. Work on the distribution of this method to inhomogeneous systems of the form (1) was done M.Zh. Talipova [1].

Theorem 1. The general solution of the system (1) is represented as the sum of the general solution $\overline{Z}(x,y)$ of the corresponding homogeneous system and the particular solution $\overline{Z}_0(x,y)$ of the system (1):

$$Z(x,y)=\overline{Z}(x,y)+\overline{Z}_0(x,y). \tag{2}$$

Theorem 2. Let there be given an inhomogeneous system of Jacobi's type

$$\begin{aligned} &x\cdot(1-x)\cdot Z_{xx}+[\gamma-(\alpha+1)\,x]\cdot Z_x+n(\alpha+n)\cdot Z=\\ &\quad=7+21x+\frac{21n(n+\alpha)}{(n-1)(n+\alpha+1)}\cdot y,\\ &y\cdot(1-y)\cdot Z_{yy}+[\gamma'-(\beta+1)y]\cdot Z_y+m(\alpha+m)\cdot Z=\\ &\quad=7+21y+\frac{21m(m+\alpha)}{(m-1)(m+\alpha+1)}\cdot x.\end{aligned} \tag{3}$$

Then, the general solution of system (3) is represented as the sum of the general solution $\overline{Z}(x,y)$ of the of the corresponding homogeneous system and the particular solution $\overline{Z}_0(x,y)$ of the inhomogeneous system (3), that is

$$\begin{aligned} Z(x,y)=\overline{Z}(x,y)+\overline{Z}_0(x,y)=C_1Z_{10}(x)Z_{01}(y)+C_2Z_{10}(x)Z_{02}(y)+\\ +C_3Z_{20}(x)Z_{01}(y)+C_4Z_{20}(x)Z_{02}(y)+\left(\frac{7}{n(n+\alpha)}-\right.\\ \left.-\frac{21}{(n-1)(n+\alpha+1)}+\frac{21x}{(n-1)(n+\alpha+1)}+\frac{21y}{(n-1)(n+\alpha+1)}\right).\end{aligned}$$

Keywords: system, particular solution, homogeneous, inhomogeneous, general solution, polynomial, orthogonal.

2010 Mathematics Subject Classification: 34K29, 60H10

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Numerical Modeling of a Liquid Droplet Surface Advection: Some Tests of the Modified VOF Algorithm

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Abstract: The goals of this paper are development and validating of the original computer code based on the algorithm of the Volume-Of-Fluid method [1], which will be the part of the software project for numerical simulation of droplet surface dynamics under the influence of external physical fields (electromagnetic field, in particular). The developed version of the computer code is devoted for tracking of liquid droplet surface in the given velocity field.

To verify the efficiency of the code developed and to estimate the accuracy of the droplet surface representation a series of tests was performed. In the first test series, initially the circular and square liquid regions moved at a constant speed along one of the coordinate axes and along the diagonal. In the second test series, the circular and square liquid regions were placed in a circular velocity field and rotated about 360 degrees around the center of rotation as a solid body. In the third test series a solenoidal velocity field was used in which the initially circular shape of the droplet is stretched and twisted into a spiral around the center of rotation, that is the topology of the liquid droplet has been changed. After a while after the calculation begins, the direction of rotation is reversed and the deformed droplet returns to its original position.

The numerical simulation results shows that in all the tests the original droplet form is preserved with a sufficient degree of accuracy. In the first and second series of experiments the deviation is of the order of 1%, and in the third series the deviation does not exceed 5%. Thus, the computer code developed adequately simulates the position of the droplet surface.

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Keywords: Droplet dynamics, Volume-of-Fluid method, numerical modeling

2010 Mathematics Subject Classification: 76T99, 65Z05

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Two-dimensional calculations of stratified turbulent flow in a pipe

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Abstract: In this paper, we consider the stratified turbulent flow of a two-phase medium in inclined pipes. Based on the new turbulence model [1], a program code for calculating two-dimensional flows for the study of two-phase stratified flows in pipes was developed, including taking into account the rough of the pipeline wall. The technique for calculating two-phase flows in extended pipelines is described. The problem of stationary stratified two-phase flow in a pipe of constant cross section in the case of turbulent regime is numerically solved. Calculations of the resistance of a rough pipe are carried out and the results on the influence of roughness on pipe resistance and velocity distribution are presented.

Keywords: stratified turbulent flow, resistance, two-dimensional calculations, rough surface

47.55.Ca, 2010 Mathematics Subject Classification: 76T10

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Solution of a linear problem of Florin type with the small parameters for the system of the parabolic equations

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There is studied a linearized two phase free boundary problem of Florin type with the small parameters for the system of the parabolic equations in the Hölder spaces.

The original nonlinear free boundary problem of Florin type describes the process of filtration of liquids and gases in the porous medium.

There are proved the existence, uniqueness of the solution and uniform coercive estimates of the solution with respect to the small parameters in the Hölder spaces.

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Keywords: Parabolic equations, small parameters, coercitive estimates, Hölder space.

2010 Mathematics Subject Classification: 35K20, 35B45, 35B30, 35C15, 35R35

On construction of the set of comparison functions of the program motion in the probable statement

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Abstract: The set of first-order Ito stochastic differential equations and set of comparison functions are constructed. There is stability in probability of the given program motion with respect to comparison functions.

Assume that the set

$$\Lambda : \lambda \equiv y - \varphi(t) = 0,\ y \in R^n,\ \varphi \in C^1,\ \|\varphi\| \leq l, \tag{1}$$

is given. It is necessary to construct the corresponding set of Ito stochastic differential equations

$$\dot{y} = Y(y,t) + \sigma(y,t)\dot{\xi},\ \ \xi \in R^k, \tag{2}$$

in the class of equations admitting the existence and uniqueness of solution of the equation (2) for the initial conditions $y|_{t=t_0} = \varphi(t_0)$. And also we need to construct the set of s-dimensional vector-functions $Q(y)$. With respect to the components of $Q(y)$ there is stability in the probability of the set (1).

This stochastic problem is solved by both the Lyapunov functions method [1] and the quasi-inversion method [2].

The given statement of the problem for $\sigma \equiv 0$ is considered in [3, 4].

Keywords: stochastic differential equation, stability in probability, comparison function

2010 Mathematics Subject Classification: 34K29, 60H10 **References:**

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Autocorrelation and intercorrelation spectrum to predict of atrial fibrillation recurrence

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Abstract: Atrial fibrillation (AF) is the most common heart rhythm disorder in the world [1] and it is characterized by an uncoordinated atrial electrical activation, whose foci in most cases are found in the pulmonary veins [2].

Frequency domain analysis has been applied to atrial electrograms to quantify the temporal and spatial organization during AF in order to predict the outcome of the three months after a successful ablation intervention.

Intra-atrial recordings and pulmonary vein electrogram recordings were obtained before AF ablation procedure were analyzed by spectrum autocorrelation and intercorrelation of atrial activations from dipoles located along different atrial areas to to quantify temporal and spatial organization during AF.

Results showed changes in spectral characteristics with higher dispersion in spatial organization along the pulmonary veins in patients that had recurrence in the arrhythmia 0.14 ± 0.01 vs. 0.07 ± 0.02 compared with patients that maintained sinus rhythm ($p<0.001$).

The proposed analysis could be useful for a better understanding of electrophysiological mechanisms during AF in both groups of patients, where the electrical activity is collected from electrodes located in the pulmonary veins area before the isolation procedure.

Keywords: spectrum; correlation; atrial fibrillation; ablation **References:**

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